Online Appendix: Foreign Exchange Interventions and Intermediary Constraints

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Abstract

This Online Appendix provides additional materials to support the analysis presented in "Foreign Exchange Interventions and Intermediary Constraints." It includes theoretical derivations, proofs, and further analysis of the Gamma-Eta model, as well as supplementary descriptions of our data, additional robustness checks, and further empirical analyses of foreign exchange intervention by the BCB.

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Contents

A Theory

In this section of the Online Appendix, we provide details of the Gamma-Eta model presented in Section 2 of the main paper. We provide a detailed structure of the model and its key assumptions in Figure A.1, followed by a formal derivation of the key equilibrium conditions in Section A.2. We linearize the model in Section A.3 and solve for the real exchange rate in Section A.4. We provide a brief discussion of exchange rate dynamics in Section A.5. We prove the model's key predictions in Section A.6. Finally, we discuss sterilized interventions in Section A.7, and swap interventions in Section A.8.

A.1 Home Household Problem

The Lagrangian formulation of the Home Household's intertemporal objective function for $t = 0$ is given by

$$\begin{aligned}
\mathcal{L}_0 = \sum_{s=0}^{2} \beta^s \, \mathrm{E}_0^{(-)}[\chi_s \ln C_{NTt} + \alpha_s \ln C_{Ht} + \iota_s \ln C_{Fs}] \\
+\beta^0 \lambda_0 \, \mathrm{E}_0^{(-)}[Y_{NT0} + p_{H0} Y_{H0} - C_{NT0} - p_{H0} C_{H0} - p_{F0} C_{F0} - Q_{H0}] \\
+\beta^1 \lambda_1 \, \mathrm{E}_1^{(-)}[Y_{NT1} + p_{H1} Y_{H1} - C_{NT1} - p_{H1} C_{H1} - p_{F1} C_{F1} - Q_{H1} + R_{H1} Q_{H0}] \\
+\beta^2 \lambda_2 \, \mathrm{E}_2^{(-)}[Y_{NT2} + p_{H2} Y_{H2} - C_{NT2} - p_{H2} C_{H2} - p_{F2} C_{F2} + R_{H2} Q_{H1}] \,,
\end{aligned} \tag{1}$$

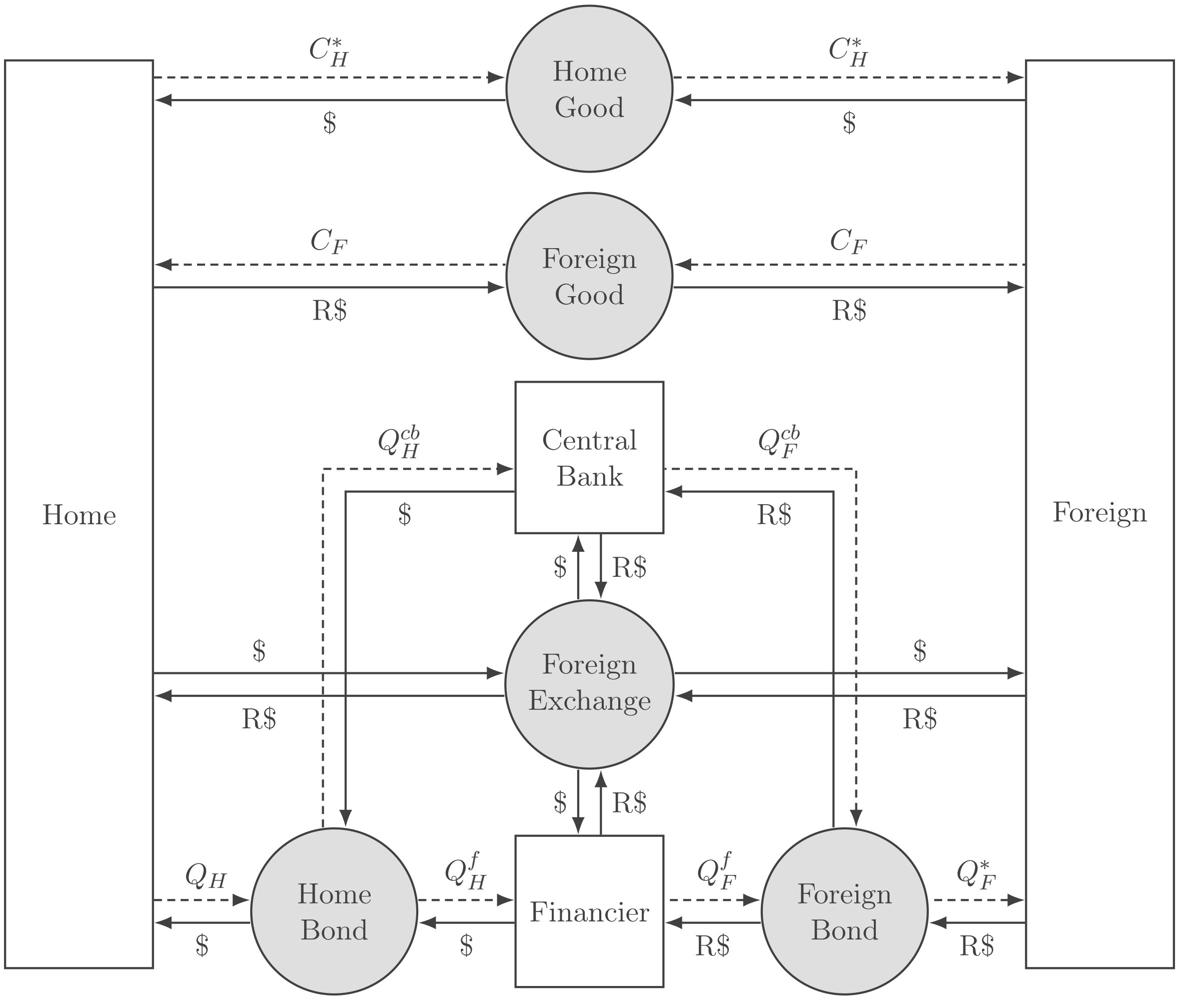
and derivatives $\partial \mathcal{L}_0 / \partial C_{NTs}$ and $\partial \mathcal{L}_0 / \partial Q_{Hs}$ lead to the optimality conditions in (3) for $s < 2$. Similar Lagrangian formulations can be written for the Household's intertemporal problems in $t = 1, 2$.

The Home Household's intratemporal Lagrangian is given by

$$\mathcal{L}'_t = \chi_s \ln C_{NTt} + \alpha_s \ln C_{Ht} + \iota_s \ln C_{Fs} + \lambda'_t (p_t C_t - C_{NTt} - p_{Ht} C_{Ht} - p_{Ft} C_{Ft}) \,, \tag{2}$$

and combining the derivative $\partial \mathcal{L}'_t / \partial C_{NTt}$ with the derivatives $\partial \mathcal{L}'_t / \partial C_{Ht}$ and $\partial \mathcal{L}'_t / \partial C_{Ft}$ yields the optimality conditions in (5).

Figure A.1: Real and financial flows in the two-country model



Notes. The figure depicts trade flows resulting from a positive trade shock $\Delta\iota_t$ that induces the Home Household to sell Home bonds and the Financier and Central Bank to buy Home bonds. Decision makers are depicted as rectangles, markets are depicted as circles, and trade flows are depicted as arrows. An arrow into a market depicts a supply flow, and an arrow out of a market depicts a demand flow. Real flows are shown as dashed lines, currency flows are shown as solid lines with labels "$" for USD and "R$" for BRL. Currency operates in the background of our economy, as a medium of exchange only, in zero net supply. Within-country goods trade, as well as profit or loss transfers from the Financier and the Central Bank to the Foreign Household are omitted.

A.2 Foreign Household Problem

The Foreign Household solves an intertemporal utility maximization problem,

$$\max_{\left\{C^*_{NTs},C^*_{Hs},C^*_{Fs}\right\}_{s=t}^{2}} \sum_{s=t}^{2} \beta^{*s}\, \mathrm{E}_t^{(\cdot)}[\varphi^*_s \ln C^*_s]\,, \tag{3}$$

where $t \in \{0, 1, 2\}$, β^* denotes the Foreign Household's subjective discount factor, $\varphi_t \equiv \chi_t^* + \iota_t^* + \alpha_t^*$ denotes a sum of stochastic preference shocks, and where

$$C_t^* \equiv \left[(C_{NTt}^*)^{\chi_t^*} (C_{Ht}^*)^{\iota_t^*} (C_{Ft}^*)^{\alpha_t^*} \right]^{\frac{1}{\varphi_t^*}} \tag{4}$$

denotes the Foreign Household consumption basket, a sub-utility function composed of Foreign non-tradable goods C_{NTt}^*, Home tradable goods C_{Ht}^*, and Foreign tradable goods C_{Ft}^*. The intertemporal maximization is subject to period budget constraints of the form

$$Q_{Ft}^* + p_{Ht}^* C_{Ht}^* + p_{Ft}^* C_{Ft}^* + C_{NTt}^* = Y_{NTt}^* + p_{Ft}^* Y_{Ft}^* + R_{Ft}^* Q_{Ft-1}^* + \Pi_t^{f*} + \Pi_t^{cb*} \,, \tag{5}$$

where Q_{Ft}^* denotes the real value of Foreign holdings of the Foreign bond, with $Q_{F-1}^* = Q_{F2}^* = 0$, where p_{Ht}^* and p_{Ft}^* denote relative prices of the Home and Foreign tradable goods, where Y_{NTt}^* and Y_{Ft}^* denote stochastic endowments of the Foreign non-tradable and tradable goods, respectively, and where Π_t^{f*} and Π_t^{cb*} denote real profits transferred to the Foreign Household by the Financier and the Central Bank, respectively. The Foreign non-tradable good is the numéraire for the Foreign economy.

The Lagrangian formulation of the Foreign Household's intertemporal objective function for $t = 0$ is given by

$$\begin{aligned}
\mathcal{L}_0^* = \sum_{s=0}^{2} & \beta^{*s} \, \mathrm{E}_0^{(-)} [\chi_s^* \ln C_{NTt}^* + \iota_s^* \ln C_{Ht}^* + \alpha_s^* \ln C_{Fs}^*] \\
& + \beta^{*0} \lambda_0^* \, \mathrm{E}_0^{(-)} [Y_{NT0}^* + p_{F0}^* Y_{F0}^* - C_{NT0}^* - p_{H0}^* C_{H0}^* - p_{F0}^* C_{F0}^* - Q_{H0}] \\
& + \beta^{*1} \lambda_1^* \, \mathrm{E}_1^{(-)} [Y_{NT1}^* + p_{F1}^* Y_{F1}^* - C_{NT1}^* - p_{H1}^* C_{H1}^* - p_{F1}^* C_{F1}^* - Q_{H1}^* + R_{F1}^* Q_{F0}^*] \\
& + \beta^{*2} \lambda_2^* \, \mathrm{E}_2^{(-)} [Y_{NT2}^* + p_{F2}^* Y_{F2}^* - C_{NT2}^* - p_{H2}^* C_{H2}^* - p_{F2}^* C_{F2}^* + R_{F2}^* Q_{F1}^*] \,,
\end{aligned} \tag{6}$$

and derivatives $\partial \mathcal{L}_0^* / \partial C_{NTs}^*$ and $\partial \mathcal{L}_0^* / \partial Q_{Fs}^*$ lead to the following optimality conditions,

$$\mathrm{E}_t^{(-)} [\lambda_s^*] = \mathrm{E}_t^{(-)} \left[\chi_s^* / C_{NTs}^* \right] \quad \text{and} \quad \mathrm{E}_t^{(-)} [\lambda_s^*] = \mathrm{E}_t^{(-)} \left[\lambda_{s+1}^* R_{Fs+1}^* \right] , \tag{7}$$

where λ_s^* denotes the Lagrange multiplier on the Foreign Household's period-s budget constraint and where R^*_{Fs+1} denotes the gross real return on the Foreign bond, with $t \leq s < 2$.

The Foreign Household also solves an intratemporal utility maximization problem, allocating expenditure across domestic and international goods. Specifically, the Foreign Household maximizes the logarithm of its sub-utility function in (4),

$$\max_{C^*_{NTt}, C^*_{Ht}, C^*_{Ft}} \chi^*_t \ln C^*_{NTt} + \iota^*_t \ln C^*_{Ht} + \alpha^*_t \ln C^*_{Ft} \,, \tag{8}$$

subject to a consumption expenditure constraint,

$$p^*_t C^*_t = C^*_{NTt} + p^*_{Ht} C^*_{Ht} + p^*_{Ft} C^*_{Ft} \,, \tag{9}$$

where p^*_t is the Foreign price index in terms of the Foreign non-tradable numéraire, and where the Foreign Household takes total consumption expenditure $p^*_t C^*_t$ as fixed in the intratemporal problem.

The Lagrangian formulation of the Foreign Household's intratemporal objective function is given by

$$\mathcal{L}'^*_t = \chi^*_t \ln C^*_{NTt} + \iota^*_t \ln C^*_{Ht} + \alpha^*_t \ln C^*_{Ft} + \lambda'^*_t (p^*_t C^*_t - C^*_{NTt} - p^*_{Ht} C^*_{Ht} - p^*_{Ft} C^*_{Ft}) \,, \tag{10}$$

and combining the derivative $\partial \mathcal{L}'^*_t / \partial C^*_{NTt}$ with the derivatives $\partial \mathcal{L}'^*_t / \partial C^*_{Ht}$ and $\partial \mathcal{L}'^*_t / \partial C^*_{Ft}$ yields the following optimality conditions for consumption expenditure on Home and Foreign tradable goods,

$$p^*_{Ft} C^*_{Ft} = (\chi^*_t / C^*_{NTt}) \alpha^*_t \quad \text{and} \quad p^*_{Ht} C^*_{Ht} = (\chi^*_t / C^*_{NTt}) \iota^*_t \,. \tag{11}$$

As we did for the Home Household, we make two simplifying assumptions for the Foreign Household: the Foreign non-tradable endowment adjusts proportionally to fluctuations in the Foreign preference for non-tradable goods, $Y^*_{NTt} = \chi^*_t$, and the Foreign Household does not discount future utility, $\beta^* = 1$.

A.3 Deviations from Steady State

To solve the model, we write key equations in terms of deviations from the non-stochastic steady state. We first derive the steady-state equilibrium, and then derive the expressions in deviations from the steady state for the bond demands and bond market clearing stated in (12)–(15) in the main paper.

Steady State. To indicate non-stochastic steady-state values, we omit time subscripts from variables. Under our simplifying assumptions, the Household optimality conditions in (3) and (7) imply that steady-state gross real returns on Home and Foreign bonds equal one,

$$R_H = R_F^* = 1\,. \tag{12}$$

The Financier's value function (6) is linear in the Financier's bond holdings, so the Financier's credit constraint (7) holds with equality. Using the Financier's value function, credit constraint, and balance sheet constraint (8), we obtain the optimality condition in (9). The Financier's optimality condition in (9) then implies zero steady-state bond holdings for the Financier.

The Central Bank's steady-state bond holdings in (10) depend on the steady-state value of the trade shock and the steady-state values of the Central Bank's intervention thresholds. We assume that preferences for imports equal one in the non-stochastic steady state, $\iota = \iota^* = 1$, which implies a steady-state trade shock of zero, $\Delta\iota = 0$. We assume the Central Bank sets its intervention thresholds to zero in the non-stochastic steady-state, $\underline{\iota} = \bar{\iota} = 0$. The Heaviside function is defined to take a value of zero when its argument is zero, so $\Delta H = H(\Delta\iota - \bar{\iota}) - H(\underline{\iota} - \Delta\iota) = 0$. The Central Bank's steady state bond holdings are therefore zero.

Bond market clearing then implies zero bond holdings for the Home Household, so that

$$Q_H = Q_H^f = Q_H^{cb} = 0\,. \tag{13}$$

Market clearing for Home tradable and non-tradable goods requires that

$$Y_{Ht} = C_{Ht} + C^{*}_{Ht} \quad \text{and} \quad Y_{NTt} = C_{NTt}\,. \tag{14}$$

Combining goods market clearing conditions in (14) with the Home Household period budget constraint in (2) evaluated at the non-stochastic steady state, using goods expenditures in (5) and (11) with our simplifying assumptions, using steady-state bond holdings in (13), and using the Law of One Price for the Home tradable good ($p_{Ht} = e_t p^{*}_{Ht}$), we obtain the following steady state real exchange rate,

$$e = 1\,, \tag{15}$$

where we have used $\iota = \iota^{*} = 1$. The steady-state results in (12), (13), and (15) suffice for the derivations of bond demands that we turn to next.

Bond Demands. We begin by deriving the Home Household's Home bond demand in (13). The Home Household's period zero budget constraint can be rewritten using market clearing conditions for the Home tradable and non-tradable in (14), the Law of One Price for the Home tradable good, goods expenditures in (5) and (11), and our simplifying assumptions on preferences to obtain

$$Q_{H0} + \iota_0 = e_0 \iota^{*}_0\,.$$

Linearizing around the non-stochastic steady state, in logs with respect to e_t and ι_t, and levels with respect to Q_{H0}, we obtain

$$(Q_{H0} - Q_H) + \iota\hat{\iota}_0 = e\iota^{*}\hat{e}_0 + e\iota^{*}\hat{\iota}^{*}_0 + O\big(\epsilon^2\big)\,,$$

where we have used $Q_H = 0$, and where we define $\hat{e}_t \equiv \ln e_t - \ln e$, $\hat{\iota}_t \equiv \ln \iota_t - \ln \iota$ and $\hat{\iota}^{*}_t \equiv \ln \iota^{*}_t - \ln \iota^{*}$. Letting $\hat{Q}_{Ht} \equiv (Q_{Ht} - Q_H)/\iota$ and $\Delta\hat{\iota}_t \equiv \hat{\iota}_t - \hat{\iota}^{*}_t$, and using $e = \iota = \iota^{*} = 1$, we obtain

$$\hat{Q}_{H0} = \hat{e}_0 - \Delta\hat{\iota}_0 + O\big(\epsilon^2\big)\,. \tag{16}$$

Following a similar procedure for the period one budget constraint, we first obtain

$$Q_{H1} + \iota_1 = e_1\iota_1^* + R_{H1}Q_{H0}\,,$$

which we linearize around the non-stochastic steady state to obtain

$$\begin{aligned}&(Q_{H1} - Q_H) + \iota\hat{\iota}_1 = e\iota^*\hat{e}_1 + e\iota^*\hat{\iota}_1^* + R_H Q_H \hat{R}_{H1} + R_H(Q_{H0} - Q_H) + O\big(\epsilon^2\big)\\ \Leftrightarrow\quad &\hat{Q}_{H1} = \hat{e}_1 - \Delta\hat{\iota}_1 + \hat{Q}_{H0} + O\big(\epsilon^2\big)\,,\end{aligned}$$

where $\hat{R}_{Ht} \equiv \ln R_{Ht} - \ln R_H$, and where we have used $e = \iota = \iota^* = 1$ and $Q_H = 0$. We combine this linearized expression with (16) to obtain

$$\hat{Q}_{H1} = \hat{e}_0 + \hat{e}_1 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 + O\big(\epsilon^2\big)\,. \tag{17}$$

For the period two budget constraint, we first obtain

$$\iota_2 = e_2\iota_2^* + R_{H2}Q_{H1}\,,$$

which we linearize around the non-stochastic steady state to obtain

$$\begin{aligned}&\iota\hat{\iota}_2 = e\iota^*\hat{e}_2 + e\iota^*\hat{\iota}_2^* + R_H Q_H \hat{R}_{H2} + R_H(Q_{H1} - Q_H) + O\big(\epsilon^2\big)\\ \Leftrightarrow\quad &0 = \hat{e}_2 - \Delta\hat{\iota}_2 + \hat{Q}_{H1} + O\big(\epsilon^2\big)\,,\end{aligned}$$

where we have used $e = \iota = \iota^* = 1$ and $Q_H = 0$. We combine this linearized expression with (17) to obtain

$$0 = \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 - \Delta\hat{\iota}_2 + O\big(\epsilon^2\big)\,. \tag{18}$$

The expressions in (16)–(18) appear in (13) in the main paper.

The Financier's linearized Home bond demand derives from the Financier's optimality condition in (9). We linearize (9) as follows,

$$\frac{1}{e}\hat{Q}^f_{Ht} - \frac{Q^f_H}{e}\,\mathrm{E}^{(-)}_t[\hat{e}_t] = \frac{1}{\Gamma}\,\mathrm{E}^{(-)}_t\left[R_H\hat{R}_{Ht+1} + R^*_F\left(\hat{e}_{t+1} - \hat{e}_t + \hat{R}^*_{Ft+1}\right)\right] + O\left(\epsilon^2\right), \tag{19}$$

where we define $\hat{Q}^f_{Ht} \equiv (Q^f_{Ht} - Q^f_H)/\iota$. This expression simplifies because $Q^f_H = 0$ and $e = R_H = R^*_F = 1$, and because $\mathrm{E}^{(-)}_t\left[\hat{R}_{Ht+1}\right] = \mathrm{E}^{(-)}_t\left[\hat{R}^*_{Ft+1}\right] = 0$. The latter result derives from linearizations of the Home and Foreign Household intertemporal optimality conditions in (5) and (11). Linearizing (5), assuming $\chi_t = C_{NTt}$ and $\beta = 1$,

$$\mathrm{E}^{(-)}_t\left[\hat{\lambda}_s\right] = \mathrm{E}^{(-)}_t\left[\hat{\chi}_s - \hat{C}_{NTs}\right] = 0 + O\left(\epsilon^2\right) \quad \Rightarrow \quad \mathrm{E}^{(-)}_t\left[\hat{R}_{Hs+1}\right] = 0 + O\left(\epsilon^2\right),$$

and linearizing (11), assuming $\chi^*_t = C^*_{NTt}$ and $\beta^* = 1$,

$$\mathrm{E}^{(-)}_t\left[\hat{\lambda}^*_s\right] = \mathrm{E}^{(-)}_t\left[\hat{\chi}^*_s - \hat{C}^*_{NTs}\right] = 0 + O\left(\epsilon^2\right) \quad \Rightarrow \quad \mathrm{E}^{(-)}_t\left[\hat{R}^*_{Fs+1}\right] = 0 + O\left(\epsilon^2\right).$$

These results combine with (19), using $Q^f_H = 0$ and $e = R_H = R^*_F = 1$, to yield the Financier's linearized Home bond demand in (14).

The Central Bank's bond demand written in deviations from steady state derives from the policy rule in (10). We can rewrite the left-hand side of (10) as follows,

$$Q^{cb}_{Ht} = Q^{cb}_{Ht} - Q^{cb}_H = (Q^{cb}_{Ht} - Q^{cb}_H)/\iota = \hat{Q}^{cb}_{Ht}, \tag{20}$$

where we define $\hat{Q}^{cb}_{Ht} \equiv (Q^{cb}_{Ht} - Q^{cb}_H)/\iota$ and use $Q^{cb}_H = 0$ and $\iota = 1$. We leave the right-hand side of the Central Bank's policy rule unchanged, and use (20) to obtain the linearized policy rule in (15).

Finally, we write the Home bond market clearing condition in deviations from steady state using the definitions of $\hat{Q}_{Ht}$, $\hat{Q}^f_{Ht}$, and $\hat{Q}^{cb}_{Ht}$ stated above. The expression (12) is exact, given that bond holdings are linearized in levels.

A.4 Real Exchange Rate Solutions

To solve for the real exchange rate, we combine the linearized bond demands of the Home Household in (13), the Financier in (14), and the Central Bank in (15) with the market clearing condition in (12) to obtain a system of three linear equations,

$$0 = \hat{e}_0 - \Delta\hat{\imath}_0 + q\Delta H_0 + \frac{1}{\Gamma}\,\mathrm{E}_0^{(-)}[\hat{e}_0 - \hat{e}_1] + O\big(\epsilon^2\big) \tag{21}$$

$$0 = \hat{e}_0 + \hat{e}_1 - \Delta\hat{\imath}_0 - \Delta\hat{\imath}_1 + q\Delta H_1 + \frac{1}{\Gamma}\,\mathrm{E}_1^{(-)}[\hat{e}_1 - \hat{e}_2] + O\big(\epsilon^2\big) \tag{22}$$

$$0 = \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\imath}_0 - \Delta\hat{\imath}_1 - \Delta\hat{\imath}_2 + O\big(\epsilon^2\big)\,, \tag{23}$$

in the endogenous real exchange rates $\hat{e}_0$, $\hat{e}_1$, and $\hat{e}_2$ and their conditional expected values. Because conditional expected values appear in the system of three equations, we need additional conditions to pin down equilibrium real exchange rates.

Taking the expectations of (21)–(23) conditional on period-zero information and the expectations of (22) and (23) conditional on period-one information, we obtain five additional conditions, which gives us a system of eight linear equations in total and allows us to solve for $\hat{e}_0$, $\hat{e}_1$, $\hat{e}_2$, $\mathrm{E}_0^{(-)}[\hat{e}_0]$, $\mathrm{E}_0^{(-)}[\hat{e}_1]$, $\mathrm{E}_0^{(-)}[\hat{e}_2]$, $\mathrm{E}_1^{(-)}[\hat{e}_1]$, and $\mathrm{E}_1^{(-)}[\hat{e}_2]$. Standard methods can be used to solve the system. We omit the intermediate algebra and provide final solutions for realized real exchange rates,

$$\begin{aligned}\hat{e}_0 = \Delta\hat{\imath}_0 &- \frac{2\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_1] - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2]}{3+\Gamma} - \frac{\Gamma\big(\mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2 - \Delta\hat{\imath}_0]\big)}{(1+\Gamma)(3+\Gamma)} \\ &- q\frac{\Delta H_0 - \eta_{0|0}}{1+\Gamma} - \Gamma q\frac{\Delta H_0}{1+\Gamma} - \Gamma q\frac{\eta_{1|0} - \eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\Big(\epsilon^2\Big)\,,\end{aligned} \tag{24}$$

$$\begin{aligned}\hat{e}_1 = \Delta\hat{\imath}_1 &- \frac{2\Delta\hat{\imath}_1 - \Delta\hat{\imath}_0 - \mathrm{E}_1^{(-)}[\Delta\hat{\imath}_2]}{3+\Gamma} + \frac{(1+\Gamma)\big(\Delta\hat{\imath}_1 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_1]\big)}{(2+\Gamma)(3+\Gamma)} + \frac{\mathrm{E}_1^{(-)}[\Delta\hat{\imath}_2] - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2]}{(2+\Gamma)(3+\Gamma)} \\ &- q\frac{\Delta H_1 - \eta_{1|1}}{2+\Gamma} - q\frac{(\Delta H_1 - \eta_{1|1}) - (\Delta H_0 - \eta_{0|0})}{2+\Gamma}\,, \\ &- \Gamma q\frac{\Delta H_1 - \Delta H_0}{3+\Gamma} - \Gamma q\frac{(\Delta H_1 - \eta_{1|0}) - (\Delta H_0 - \eta_{0|0})}{(2+\Gamma)(3+\Gamma)} + O\Big(\epsilon^2\Big)\,, \quad \text{and}\end{aligned} \tag{25}$$

$$
\begin{aligned}
\hat{e}_2 = \Delta\hat{\iota}_2 &- \frac{2\Delta\hat{\iota}_2 - \Delta\hat{\iota_0} - \Delta\hat{\iota_1}}{3+\Gamma} + \frac{\Gamma(\Delta\hat{\iota}_2 - \Delta\hat{\iota}_0)}{(1+\Gamma)(3+\Gamma)} + \frac{2\Delta\hat{\iota}_2 - \mathrm{E}_0^{(-)}[\Delta\iota_2] - \mathrm{E}_1^{(-)}[\Delta\hat{\iota}_2]}{(1+\Gamma)(3+\Gamma)} \\
&+ \frac{\Gamma\left(\Delta\hat{\iota}_2 - \mathrm{E}_1^{(-)}[\Delta\iota_2]\right)}{(1+\Gamma)(3+\Gamma)} + \frac{\Delta\hat{\iota}_1 - \mathrm{E}_0^{(-)}[\Delta\hat{\iota}_1]}{(2+\Gamma)(3+\Gamma)} - \frac{\mathrm{E}_1^{(-)}[\Delta\iota_2] - \mathrm{E}_0^{(-)}[\Delta\iota_2]}{(2+\Gamma)(3+\Gamma)} \\
&+ q\frac{\Delta H_0 - \eta_{0|0}}{1+\Gamma} + \Gamma q\frac{\Delta H_0}{1+\Gamma} + \Gamma q\frac{\eta_{1|0} - \eta_{0|0}}{(1+\Gamma)(3+\Gamma)} \\
&+ q\frac{\Delta H_1 - \eta_{1|1}}{2+\Gamma} + q\frac{(\Delta H_1 - \eta_{1|1}) - (\Delta H_0 - \eta_{0|0})}{2+\Gamma} \\
&+ \Gamma q\frac{\Delta H_1 - \Delta H_0}{3+\Gamma} + \Gamma q\frac{(\Delta H_1 - \eta_{1|0}) - (\Delta H_0 - \eta_{0|0})}{(2+\Gamma)(3+\Gamma)} + O\left(\epsilon^2\right).
\end{aligned} \tag{26}
$$

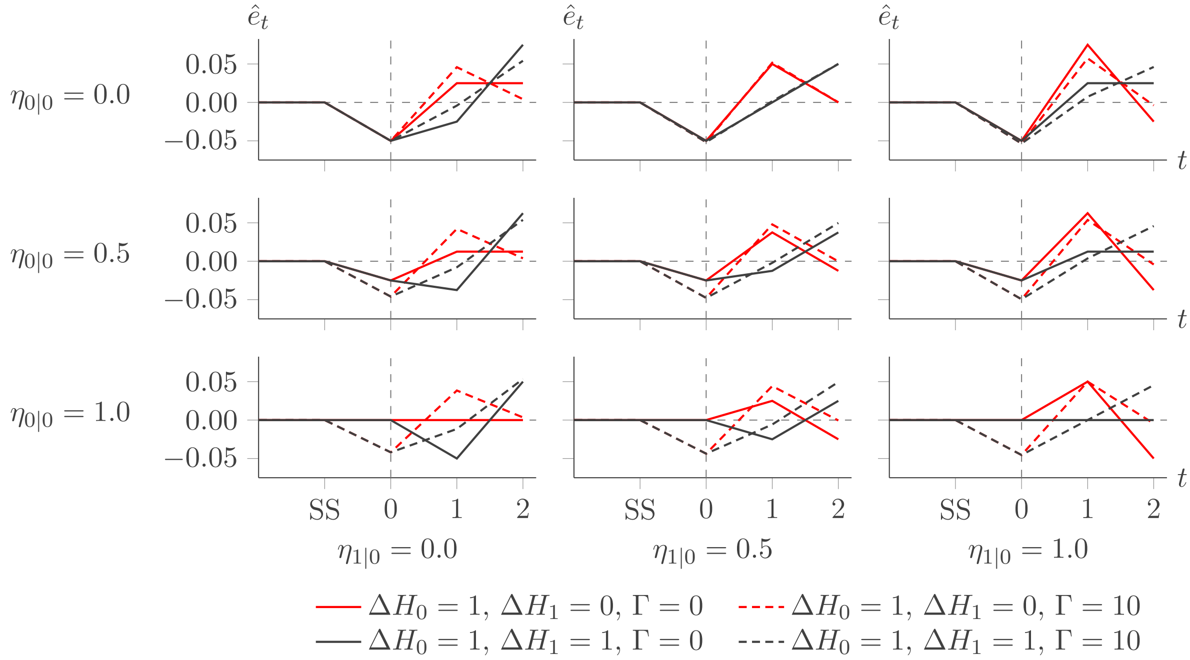
A.5 Real Exchange Rate Dynamics

We limit our theoretical model to three periods because we seek interpretable closed-form expressions that deliver intuitive qualitative predictions on the real exchange rate effects of Central Bank interventions. In our model, interventions are only possible in the first two periods, when bond trading occurs. This timeline gives us enough flexibility to differentiate between short-lived swap and long-lived spot interventions, but not more. The constrained timeline of the model precludes a nuanced study of economic dynamics; however, limited insights can still be gained from analyzing the three-period path of the real exchange rate in response to interventions.

Figure A.2 plots the path of the real exchange rate in response to positive Central Bank interventions at scale $q = 0.05$. To isolate the effects of interventions, we set trade shocks to zero, $\Delta\hat{\iota}_t = 0$. In the figure, the Central Bank intervenes positively to strengthen Home currency and lower the real exchange rate $\hat{e}_0$. In a positive intervention, the Central Bank sells Foreign bonds and buys Home bonds, exchanging Foreign currency for Home currency in the process. The Home currency demand caused by the Central Bank's bond trade strengthens Home currency and puts downward pressure on $\hat{e}_0$. The model's equation (13) produces what Gabaix and Maggiori (2015) describe as a "boomerang" effect.

Figure A.2 shows how the path of the real exchange rate depends on the type of intervention (swap versus spot), the degree of anticipation (unanticipated versus partially or fully anticipated), and the state of credit constraints (tight or loose). In the middle row, we show the empirically-relevant case where agents assign a positive probability to

Figure A.2: Real Exchange Rates Dynamics



Notes. The figure plots the path of the real exchange rate over the three periods of the model in response to Central Bank interventions at scale $q = 0.05$, beginning from a steady-state position (SS). To isolate the effects of interventions, we set trade shocks to zero, $\Delta\hat{\imath}_t = 0$. Red lines represent short-lived swap interventions. Black lines represents long-lived spot interventions. Solid lines represent loose credit constraints. Dashed lines represent tight credit constraints. Each row corresponds to a value for the probability of a positive period-zero intervention as perceived by Households and the Financier. Each column corresponds to a value for the probability of a positive period-one intervention as perceived by Households and the Financier. Probability values capture three cases: unanticipated ($\eta_{s|t} = 0$), partially anticipated ($\eta_{s|t} = 0.5$), and fully anticipated ($\eta_{s|t} = 1$).

a period zero intervention but do not fully anticipate it. In this case, the exchange rate effect is largest and most sustained when the intervention is long-lived (black) and the credit constraint is tight (dashed). When fully anticipated with loose credit constraints, interventions have no effect, as shown in the bottom left and right figures. In between these extremes, the Central Bank can shape the path of the real exchange rate in many ways by varying the type of intervention and manipulating Household and Financier expectations.

The responses above the main diagonal in Figure A.2 are excluded under the regularity conditions sign $\Delta\eta_{s|t} =$ sign $\Delta\eta_{s+1|t}$ and $|\Delta\eta_{s|t}| \geq |\Delta\eta_{s+1|t}|$ imposed in Section 2. These conditions are simple, intuitive, and sufficient for Predictions 4 and 5 (the only predictions where they are used), but stronger than necessary, as we show in the proofs.

A.6 Prediction Proofs

Proof 1 (Direction of Interventions)**.** *We now prove Prediction 1. Consider a Central Bank intervention whereby the Central Bank constructs a portfolio of Home and Foreign bonds. The real exchange rate responds to the currency flows arising from the portfolio construction as follows,*

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma \Delta H_0}{1+\Gamma} - \frac{\Delta H_0 - \mathrm{E}_0^{(-)}[\Delta H_0]}{1+\Gamma} - \frac{\Gamma\, \mathrm{E}_0^{(-)}[\Delta H_1 - \Delta H_0]}{(1+\Gamma)(3+\Gamma)}\,. \tag{27}$$

The direction of the response depends on the direction of the intervention. Recall that $\Delta H_0 \in \{-1, 0, 1\}$ *and consider two cases: 1) positive intervention, and 2) negative intervention.*

- ***Case 1:*** *Consider a* ***positive intervention****,* $\Delta H_0 = 1$*. In this case,*

$$\begin{aligned}
\frac{\partial \hat{e}_0}{\partial q} = -1 + \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &\overset{!}{<} 0 \\
\Leftrightarrow \quad \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &< 1 \\
\Leftarrow \quad \frac{3+2\Gamma}{(1+\Gamma)(3+\Gamma)} + \frac{\Gamma}{(1+\Gamma)(3+\Gamma)} &< 1 \\
\Leftrightarrow \quad \frac{3}{3+\Gamma} &< 1 \\
\Leftrightarrow \quad 0 &< \Gamma\,.
\end{aligned}$$

- ***Case 2:*** *Consider a* ***negative intervention****,* $\Delta H_0 = -1$*. In this case,*

$$\begin{aligned}
\frac{\partial \hat{e}_0}{\partial q} = 1 + \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &\overset{!}{>} 0 \\
\Leftrightarrow \quad \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &> -1 \\
\Leftarrow \quad -\frac{3+2\Gamma}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma}{(1+\Gamma)(3+\Gamma)} &> -1 \\
\Leftrightarrow \quad -\frac{3}{3+\Gamma} &< -1 \\
\Leftrightarrow \quad 0 &< \Gamma\,.
\end{aligned}$$

Hence, for $\Delta H_0 \neq 0$ and $\Gamma > 0$,

$$\operatorname{sign} \frac{\partial \hat{e}_0}{\partial q} = -\operatorname{sign} \Delta H_0 \,.$$

In words, a positive intervention strengthens Home currency and lowers the real exchange rate, while a negative intervention weakens Home currency and raises the real exchange rate.

Proof 2 (Anticipated versus Unanticipated Interventions)**.** *We now prove Prediction 2. Consider a Central Bank communication that raises the expectation of a positive intervention in period zero ($\Delta\eta_{0|0} > 0$). The second partial derivative of (27) with respect to the parameter $\eta_{0|0}$ that governs the expectation of a period zero intervention is given by*

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{0|0}} = \frac{3 + 2\Gamma}{(1+\Gamma)(3+\Gamma)} > 0 \,.$$

In words, if the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by less if Households and the Financier anticipate the intervention in period zero. If $\Gamma = 0$ and $\Delta H_0 = \eta_{0|0}$, the right-hand side of (27) equals zero as can be seen by inspection.

We obtain a second prediction in this context, which we omitted from Prediction 2 in the main paper for brevity. Consider a Central Bank communication that raises the period-zero expectation of a positive intervention in period one — e.g. to maintain an on-going spot intervention. The second partial derivative of (27) with respect to the parameter $\eta_{1|0}$ that governs the expectation of a period-one intervention is given by

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{1|0}} = -\frac{\Gamma}{(1+\Gamma)(3+\Gamma)} < 0 \,.$$

If the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by more than it otherwise would, if Households and the Financier anticipate that the intervention will be maintained in period one. These results suggest a potential role for forward guidance in the Central Bank's FXI policy.

Proof 3 (Spot versus Swap Interventions)**.** *We now prove Prediction 3. Consider four cases of fully anticipated Central Bank interventions: 1) a positive spot intervention, 2) a positive swap intervention, 3) a negative spot intervention, and 4) a negative swap intervention.*

- ***Case 1:*** *Consider a fully anticipated* ***positive spot*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = \eta_{1|0} = 1$. *In this case, from* (27),

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma}{1+\Gamma}\,.$$

- ***Case 2:*** *Consider a fully anticipated* ***positive swap*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = 1$, $\eta_{1|0} = 0$. *In this case, from* (27),

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma}{1+\Gamma} \times \frac{2+\Gamma}{3+\Gamma}\,.$$

For $\Gamma > 0$, *cases one and two imply that*

$$\left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = \eta_{1|0} = 1} < \left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = 1,\, \eta_{1|0} = 0},$$

where the subscripts indicate the values at which the derivative is evaluated.

- ***Case 3:*** *Consider a fully anticipated* ***negative spot*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = \eta_{1|0} = -1$. *In this case, from* (27),

$$\frac{\partial \hat{e}_0}{\partial q} = \frac{\Gamma}{1+\Gamma}\,.$$

- ***Case 4:*** *Consider a fully anticipated* ***negative swap*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = -1$, $\eta_{1|0} = 0$. *In this case, from* (27),

$$\frac{\partial \hat{e}_0}{\partial q} = \frac{\Gamma}{1+\Gamma} \times \frac{2+\Gamma}{3+\Gamma}\,.$$

For $\Gamma > 0$, cases three and four imply that

$$\left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = \eta_{1|0} = -1} > \left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = -1,\, \eta_{1|0} = 0},$$

where the subscripts indicate the values at which the derivative is evaluated. Hence, for $\Delta H_0 = \Delta\eta_{0|0} \neq 0$ and $\Gamma > 0$, cases one through four imply that

$$\left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0} = \Delta H_0} > \left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0} = 0}.$$

In words, a fully anticipated long-lived spot intervention ($\Delta\eta_{1|0} = \Delta H_0$) has a greater effect on the real exchange rate than a fully anticipated short-lived swap intervention ($\Delta\eta_{1|0} = 0$), for both positive and negative interventions. In the second part of the proof to Prediction 2 we present related results for the unanticipated case.

For the case where a Central Bank communication raises the period-zero expectation that an intervention will persist, see the proof to Proposition 2.

Proof 4 (Private Intermediation)**.** *We now prove Prediction 4. From the Financier's linearized bond demand in (14), using the real exchange rate solutions for periods zero and one in (24) and (25), we obtain*

$$\begin{aligned}\hat{Q}^f_{H0} = {} & \frac{\Delta\hat{\iota}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\iota}_1]}{3+\Gamma} + \frac{\Delta\hat{\iota}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\iota}_2]}{(1+\Gamma)(3+\Gamma)} \\ & - q\left[\frac{\Delta\eta_{0|0}}{1+\Gamma} - \frac{\Gamma(\Delta\eta_{1|0} - \Delta\eta_{0|0})}{(1+\Gamma)(3+\Gamma)}\right] + O(\epsilon^2)\end{aligned}$$

Taking the derivative of $\hat{Q}^f_{H0}$ with respect to q, we obtain

$$\frac{\partial \hat{Q}^f_{H0}}{\partial q} = \frac{\Gamma\Delta\eta_{1|0}}{(1+\Gamma)(3+\Gamma)} - \frac{(3+2\Gamma)\Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)}.$$

This derivative takes the opposite sign of the expected intervention if

$$\frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} < 2 + \frac{3}{\Gamma}, \tag{28}$$

which holds for any $\Gamma > 0$ as long as $\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$ (which is implied by the regularity conditions we have imposed in the model primitives of Section 2).

Proof 5 (Credit Constraints)**.** *We now prove Prediction 5. From the Financier's bond demand in (14) and the real exchange rate solutions in (24) and (25),*

$$\begin{aligned} \hat{Q}^{f}_{H0} = \frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_1]}{3+\Gamma} &+ \frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2]}{(1+\Gamma)(3+\Gamma)} \\ - q\frac{\Delta\eta_{0|0}}{1+\Gamma} &+ \Gamma q \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\big(\epsilon^2\big)\,. \end{aligned} \tag{29}$$

Suppose the Central Bank is passive ($q = 0$) and consider a tightening of credit constraints,

$$\frac{\partial \hat{Q}^{f}_{H0}}{\partial \Gamma} = -\frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_1]}{(3+\Gamma)^2} - 2(2+\Gamma)\frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2]}{(1+\Gamma)^2(3+\Gamma)^2}\,. \tag{30}$$

By inspection of (29) and (30), tighter credit constraints lower the amount of intermediation the Financier undertakes if the Central Bank is passive ($q = 0$),

$$\operatorname{sign}\frac{\partial \hat{Q}^{f}_{H0}}{\partial \Gamma} = -\operatorname{sign}\hat{Q}^{f}_{H0}\,.$$

The Financier's portfolio positions in Home and Foreign bonds shrink when credit constraints tighten, limiting the ability of Households to smooth consumption intertemporally.

Suppose the Central Bank is active ($q > 0$) and consider the impact that tighter credit constraints have on the real exchange rate effect of an intervention. From (27),

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \frac{\big[(3-\Gamma^2) - (3+\Gamma)^2\big]\Delta\eta_{0|0}}{(1+\Gamma)^2(3+\Gamma)^2} - \frac{(3-\Gamma^2)\Delta\eta_{1|0}}{(1+\Gamma)^2(3+\Gamma)^2}\,. \tag{31}$$

To establish the sign of the partial derivative in (31), we evaluate four cases.

- ***Case 1:*** *Let $\Delta\eta_{0|0} > 0$, $\Delta\eta_{1|0} > 0$, and $3 - \Gamma^2 > 0$. From (31),*

$$\begin{aligned} \frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{<} 0 \quad \Leftrightarrow \quad & \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\ \Leftrightarrow \quad & -\frac{6+6\Gamma+2\Gamma^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}}\,, \end{aligned}$$

which is satisfied under the assumptions in Case 1.

- ***Case 2:*** *Let* $\Delta\eta_{0|0} > 0$, $\Delta\eta_{1|0} > 0$, *and* $3 - \Gamma^2 < 0$. *From* (31),

$$\begin{aligned}
\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{<} 0 \quad \Leftrightarrow \quad & \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} > \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\
\Leftrightarrow \quad & \frac{(3+\Gamma)^2}{3-\Gamma^2} > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}} \\
\Leftarrow \quad & 1 > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}} \quad \Leftrightarrow \quad \Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0} \,.
\end{aligned}$$

Hence, the inequality is satisfied in Case 2 when $\Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0}$.

- ***Case 3:*** *Let* $\Delta\eta_{0|0} < 0$, $\Delta\eta_{1|0} < 0$, *and* $3 - \Gamma^2 > 0$. *From* (31),

$$\begin{aligned}
\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{>} 0 \quad \Leftrightarrow \quad & \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\
\Leftrightarrow \quad & -\frac{6 + 6\Gamma + 2\Gamma^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \,,
\end{aligned}$$

which is satisfied under the assumptions in Case 3.

- ***Case 4:*** *Let* $\Delta\eta_{0|0} < 0$, $\Delta\eta_{1|0} < 0$, *and* $3 - \Gamma^2 < 0$. *From* (31),

$$\begin{aligned}
\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{>} 0 \quad \Leftrightarrow \quad & \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} > \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\
\Leftrightarrow \quad & \frac{(3+\Gamma)^2}{3-\Gamma^2} > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}} \\
\Leftarrow \quad & 1 > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}} \quad \Leftrightarrow \quad \Delta\eta_{0|0} < \frac{1}{2}\Delta\eta_{1|0} \,.
\end{aligned}$$

Hence, the inequality is satisfied in Case 4 when $\Delta\eta_{0|0} < \frac{1}{2}\Delta\eta_{1|0}$.

From (27) *and* (31) *and Cases 1–4, under the condition that* $|\Delta\eta_{0|0}| > \frac{1}{2}|\Delta\eta_{1|0}|$ *(which is implied by the regularity conditions we have imposed in the model primitives of Section 2), and assuming the Central Bank's communications are consistent with its actions* $(\operatorname{sign} \Delta\eta_{0|0} = \operatorname{sign} \Delta\eta_{1|0} = \operatorname{sign} \Delta H_0 \neq 0)$, *we have*

$$\operatorname{sign} \frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \operatorname{sign} \frac{\partial \hat{e}_0}{\partial q} \,,$$

Table A.1: Central Bank Balance Sheet View of Sterilized Intervention.

Step	Assets	Liabilities
1. Purchase Foreign Currency	Assets in Foreign Currency (+1)	Circulating Home Currency (+1)
2. Sell Home Bonds	Assets in Home Currency (−1)	Circulating Home Currency (−1)
3. Net Effect	Assets in Foreign Currency (+1) Assets in Home Currency (−1)	Circulating Home Currency (±0)

Notes. The tables illustrate the process of sterilizing a Central Bank intervention with respect to Home currency. Step 1 shows the Central Bank purchasing Foreign currency with Home currency, increasing the Home currency in circulation. Step 2 shows the Central Bank selling Home-currency bonds to offset the rise in Home currency from Step 1. Step 3 shows the net effect of Steps 1 and 2: the Home money supply remains unchanged, while the Central Bank's Foreign currency assets increase and its Home currency assets decrease. This sterilized intervention keeps the supply of Home currency constant but alters the composition of the central bank's assets.

and tighter credit constraints amplify interventions.

A.7 Sterilized Interventions

Central banks can manage exchange rates without altering the money supply through sterilized intervention. Consider a spot purchase of Foreign currency as an example intervention. In a non-sterilized intervention, the Central Bank purchases Foreign currency with Home currency, increasing the Home money supply. To "sterilize" the impact, the Central Bank sells Home-currency bonds to absorb the excess Home currency. If the sterilization is perfect, the money supply remains constant, while the ratio of Home-currency and Foreign-currency bonds held by the public and the central bank changes.

A sterilized intervention can be viewed as a combination of two transactions. First, in the FX market, the Central Bank conducts a non-sterilized intervention by purchasing Foreign currency (or Foreign-currency bonds), issuing Home currency to fund the purchase. Second, in the money market, the Central Bank "sterilizes" the effect by selling an equivalent amount of Home bonds to absorb the initial increase in the Home money supply. The net effect of a sterilized spot purchase is neutral regarding the Home currency in circulation, but there is a portfolio change in assets, with an increase (decrease) in the share of Foreign-currency assets held by the central bank (public). Table A.1 illustrates these steps in a stylized Central Bank balance sheet. Equation (2) of the Gamma-Eta model is consistent with sterilized intervention.

A.8 Swap Interventions

In this section, we show that swap interventions involving spot and forward currency market transactions are equivalent to bond-market interventions involving zero-cost positions in risk-free Home and Foreign bonds in our model under CIP. This section also serves to derive the Financier's value function in (6).

To establish this result, we consider the nominal cash flows arising from both intervention types, derive real profits from these cash flows, and show that real profits are equal for both intervention types. In view of this result, we model swaps as zero-cost bond positions rather than modeling an additional forward market in the paper. For recent models with explicit forward markets, see De Leo et al. (2024) and Liao and Zhang (2025).

Swap Cash Flows and Profits. A swap operation for the Central Bank involves two legs: a spot leg and a forward leg. For example, the Central Bank might buy Home currency and sell Foreign currency spot at t, while simultaneously selling Home currency and buying Foreign currency forward at $t+1$.

Let $P_{NT_H t}$ denote the nominal price of the Home non-tradable good, and $P_{NT_H t}N_t$ the nominal notional principal for N_t swap contracts to exchange Home and Foreign currency at spot rate S_t and forward rate $F_{t,t+1}$, where the latter exchange rates are expressed in units of Home currency per one unit of Foreign currency. Let CF_t and M_t denote generic cash flows, with asterisks indicating Foreign currency. The Central Bank's cash flows from the swap are given below:

$$
\begin{array}{lll}
 & \text{Period } t & \text{Period } t+1 \\
\text{Cash Outflow:} & M_t^* = -P_{NT_H t} N_t S_t^{-1} & M_{t+1} = -P_{NT_H t} N_t \\
\text{Cash Inflow:} & M_t = +P_{NT_H t} N_t & M_{t+1}^* = +P_{NT_H t} N_t F_{t,t+1}^{-1} \\
\text{Total Cash Flow:} & CF_t = M_t + S_t M_t^* = 0 & CF_{t+1} = M_{t+1} + S_{t+1} M_{t+1}^* \,.
\end{array}
$$

The Central Bank's nominal profit from the swap equals

$$
\tilde{\Pi}^{cb}_{Swap,t+1} = CF_{t+1} - CF_t = \left(S_{t+1} F_{t,t+1}^{-1} - 1 \right) P_{NT_H t} N_t \,, \tag{32}
$$

where the tilde notation distinguishes nominal profit from real profit.

To determine the forward price, we use the CIP condition that requires equivalent assets denominated in different currencies to earn equal returns after hedging exchange rate risk. We consider a zero-cost position in Home and Foreign bonds, satisfying

$$
S_t P^*_{B_F t} B_{Ft} + P_{B_H t} B_{Ht} = 0 \,, \tag{33}
$$

where $P_{B_H t}$ and $P^*_{B_F t}$ are Home and Foreign bond prices, and B_{Ht} and B_{Ft} are units of Home and Foreign bonds, respectively. The bonds are risk-free, single-period assets that pay one unit of the domestic non-tradable good at maturity. The equality in (33) is established by choice of units for B_{Ht} and B_{Ft}. CIP then requires equal payoffs at maturity after hedging the foreign-currency position with a forward contract,

$$
F_{t,t+1} P^*_{NT_F t+1} B_{Ft} + P_{NT_H t+1} B_{Ht} = 0 \,,
$$

where $P^*_{NT_F t}$ is the nominal price of the Foreign non-tradable good. Rearranging,

$$
-\frac{F_{t,t+1}}{S_{t+1}} \times \underbrace{\frac{P^*_{NT_F t}}{P^*_{B_F t}}}_{\substack{|||\\ R^*_{Ft+1}}} \times \underbrace{\frac{S_{t+1} P^*_{NT_F t+1}}{P_{NT_H t+1}}}_{\substack{|||\\ e_{t+1}}} \times \underbrace{\frac{P_{NT_F t}}{S_t P^*_{NT_F t}}}_{\substack{|||\\ e_t^{-1}}} \times \underbrace{\frac{S_t P^*_{B_F t} B_{Ft}}{P_{NT_H t}}}_{\substack{|||\\ Q_{Ft}}} = \underbrace{\frac{P_{NT_H t}}{P_{B_H t}}}_{\substack{|||\\ R_{Ht+1}}} \times \underbrace{\frac{P_{B_H t} B_{Ht}}{P_{NT_H t}}}_{\substack{|||\\ Q_{Ht}}} \,.
$$

The non-standard real exchange rate definition follows Gabaix and Maggiori (2015). Equation (33) implies $Q_{Ht} + Q_{Ft} = 0$, so the CIP condition simplifies to

$$F_{t,t+1} = \frac{R_{Ht+1}}{R^*_{Ft+1}} \frac{e_t}{e_{t+1}} S_{t+1} \,. \tag{34}$$

The forward price here is expressed in terms of real rates under CIP, while the forward price in Equation (18) in the paper is expressed in terms of nominal rates and allows for CIP deviations.

Substituting forward price (34) into the Central Bank's swap profit (32) and rearranging, we obtain real swap profit as

$$\Pi^{cb}_{Swap,t+1} = \frac{\tilde{\Pi}^{cb}_{Swap,t+1}}{P_{NT_H t+1}} = \left(\frac{e_{t+1}}{e_t} \frac{R^*_{Ft+1}}{R_{Ht+1}} - 1 \right) N_t \,. \tag{35}$$

Bond Cash Flows and Profits. A bond-market operation for the Central Bank involves two positions in Home and Foreign bonds: a long position in one bond and a short position in the other. Equation (33) constitutes such a position.

The Central Bank's cash flows from a long Foreign bond position ($B_{Ft} > 0$) and an offsetting short Home bond position ($B_{Ht} < 0$) are given in the below, where we again use CF_t and M_t to denote generic cash flows, with asterisks for Foreign currency.

	Period t	Period $t+1$
Cash Outflow:	$M^*_t = -P^*_{B_F t} B_{Ft}$	$M_{t+1} = P_{NT_H t+1} B_{Ht}$
Cash Inflow:	$M_t = -P_{B_H t} B_{Ht}$	$M^*_{t+1} = P^*_{NT_F t+1} B_{Ft}$
Total Cash Flow:	$CF_t = M_t + S_t M^*_t = 0$	$CF_{t+1} = M_{t+1} + S_{t+1} M^*_{t+1}$.

The Central Bank's profit from the zero-cost long-short position is given by

$$\tilde{\Pi}^{cb}_{Bond,t+1} = CF_{t+1} - CF_t = S_{t+1} P^*_{NT_F t+1} B_{Ft} + P_{NT_H t+1} B_{Ht} \,.$$

Rearranging the profit equation, we have

$$\frac{\Pi^{cb}_{Bond,t+1}}{P_{NT_Ht+1}} = \underbrace{\frac{S_{t+1}P^*_{NT_Ft+1}}{P_{NT_Ht+1}}}_{\substack{\parallel\\ e_{t+1}}} \times \underbrace{\frac{P_{NT_Ht}}{S_tP^*_{NT_Ft}}}_{\substack{\parallel\\ e_t^{-1}}} \times \underbrace{\frac{P^*_{NT_Ft}}{P^*_{B_Ft}}}_{\substack{\parallel\\ R^*_{Ft+1}}} \times \underbrace{\frac{S_tP^*_{B_Ft}B_{Ft}}{P_{NT_H}t}}_{\substack{\parallel\\ Q_{Ft}}} - \underbrace{\frac{P_{NT_Ht}}{P_{B_H}t}}_{\substack{\parallel\\ R_{Ht}}} \times \underbrace{\frac{P_{B_Ht}B_{Ht}}{P_{NT_Ht}}}_{\substack{\parallel\\ Q_{Ht}}}.$$

Simplifying, and setting $N_t = R_{Ht+1}Q_{Ht}$ in (35), we obtain

$$\Pi^{cb}_{t+1} = \frac{\tilde{\Pi}^{cb}_{Swap,t+1}}{P_{NT_Ht+1}} = \frac{\tilde{\Pi}^{cb}_{Bond,t+1}}{P_{NT_Ht+1}} = \left(R_{Ht+1}Q_{Ht} + \frac{e_{t+1}}{e_t}R^*_{Ft+1}Q_{Ft}\right), \tag{36}$$

The right-hand side of (36) equals that of the Financier's value function in (6).

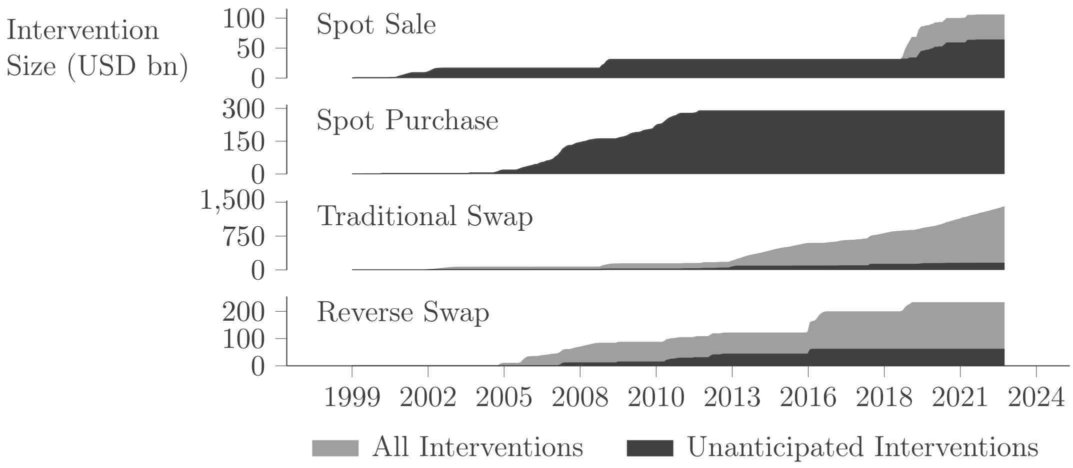
B Empirics

This section of the Online Appendix provides empirical details supporting the main findings reported in Section 4 of the paper.

We provide additional descriptive statistics for our datasets in Section B.1. We describe the relationship between Γ and exchange rate volatility in Section B.2. We tabulate the exchange rate response to unanticipated spot sell interventions at different horizons in Section B.3. We consider alternative position-based and CPI-based method of conditioning on intermediary constraints when estimating conditional exchange rate responses to FXI in Section B.4. We estimate exchange rate responses after pooling buy and sell interventions in Section B.5 for a more direct comparison with prior results reported in the FXI literature. We estimate exchange rate responses to anticipated interventions in Section B.6 and find generally weaker effects, as predicted by the Gamma-Eta Model. We estimate forward premium responses to unanticipated interventions in Section B.7, providing evidence that links CIP violations to the relative demand for currency forwards. We estimate exchange rate responses to unanticipated interventions on days with clustered interventions in Section B.8. Finally, we describe our procedure for estimating the FXI residual we use to examine the signaling channel in lower frequency data in Section B.9.

Finally, we describe our procedure for estimating the FXI residual we use to examine

Figure B.1: Cumulative BCB FXI



Notes. These figures show cumulative BCB FXI over time, by intervention type. The vertical axis shows the intervention size in USD billions, after aggregating at monthly frequency. The lighter shade shows all interventions, both anticipated and unanticipated, while the darker shade shows the subset of unanticipated interventions, defined as interventions with an announcement equal to the operation date.

the signaling channel in lower frequency data in Section B.9.

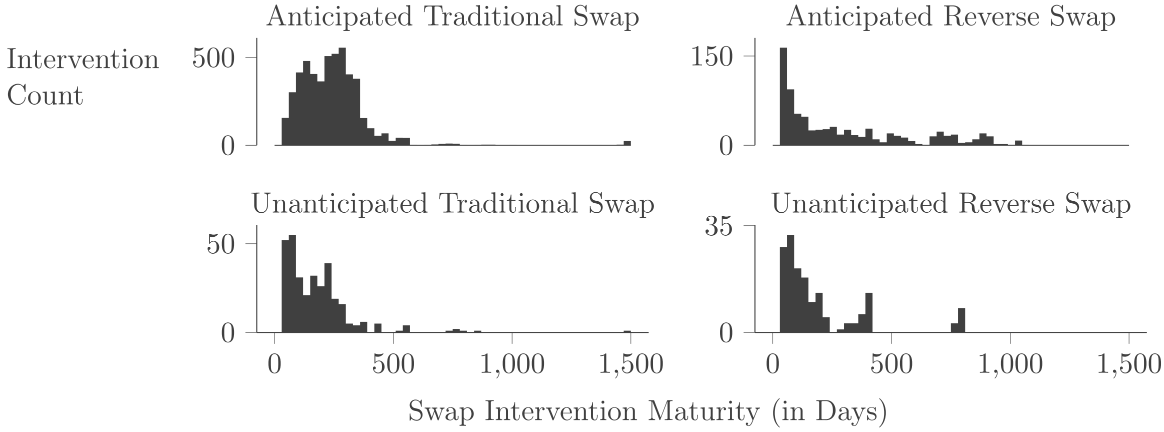
B.1 Description of Data and BCB FXI

This section of the Online Appendix presents tables and figures summarizing the BCB's FXI, as well as key statistics on the BRL/USD exchange rate and control variables from the local projections in Section 4.

Figure B.1 shows the cumulative size of BCB interventions over time, disaggregated by type (spot sales, purchases, traditional and reverse swaps), and distinguishes between anticipated and unanticipated interventions. Figure B.2 provides a histogram of swap intervention maturities, further split by traditional and reverse swaps, with the horizontal axis indicating maturity in days.

Table B.1 summarizes BCB FXI by type (spot sales/purchases and traditional/reverse swaps), showing the mean, standard deviation, and maximum expressed in USD billions, and total count for both anticipated and unanticipated interventions. Table B.2 provides summary statistics for the BRL/USD exchange rate, currency basis, and forward premium, including the mean, standard deviation, minimum, maximum, and quartiles. Table B.3 presents the same statistics for the control variables used in the local projections regressions,

Figure B.2: Maturity Breakdown of BCB Swap Interventions



Notes. These figures show 50-bin histograms of the maturities of BCB's interventions in the FX markets, by intervention type. The horizontal axes show the maturity in days. Unanticipated interventions are defined as interventions with an announcement equal to the operation date, while anticipated interventions are defined as interventions with an announcement date that precedes the operation date. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

Table B.1: Summary Statistics for BCB FXI

	Spot Sale		Spot Purchase	
	Unanticipated	Anticipated	Unanticipated	Anticipated
Mean	0.17	0.48	0.19	0.00
SD	0.22	0.39	0.24	0.00
Max	1.10	3.00	4.64	0.00
Count	385	87	1483	0.00
	Traditional Swap		Reverse Swap	
	Unanticipated	Anticipated	Unanticipated	Anticipated
Mean	0.43	0.25	0.35	0.20
SD	0.41	0.24	0.45	0.28
Max	1.85	3.50	3.38	4.00
Count	345	5094	174	846

Notes. This table shows the mean, standard deviation (SD), and maximum of BCB FXI operations, in USD billions, as well as total counts. For unanticipated interventions, the announcement date is equal to the operation date. For anticipated interventions, the announcement date precedes the operation date. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27. Mean, standard deviation, and max values are expressed in USD billion.

including the HKM intermediary capital ratio, Brazil's EMBI, US and Brazilian interest rates, market volatility, and total intervention size.

Table B.2: Summary Statistics for BRL/USD

	BRL/USD Spot Rate	1m BRL/USD	
		Currency Basis	Forward Premium
Mean	2.773	−220.106	186.341
SD	1.105	133.919	111.107
Min	1.207	−639.128	−79.337
25%	1.916	−302.106	101.390
75%	3.268	−124.770	275.808
Max	5.905	147.176	638.495

Notes. This table shows mean, standard deviation (SD), minimum, 25% percentile, 75% percentile, and maximum values for the BRL/USD spot rate, currency basis, and forward premium. The spot rate is expressed in units of BRL per USD. The currency basis and forward premium are expressed in basis points (bp). The sample period for the spot rate is from 1999-01-22 to 2023-04-27. The sample period for the currency basis and forward premium is from 2003-11-24 to 2023-04-27.

Table B.3: Summary Statistics for Control Variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	HKM	$EMBI$	i^{US}	i^{BR}	i_s^{US}	i_s^{BR}	INT^{TOL}	S^{VOL}
Mean	0.074	0.042	1.531	11.712	0.614	0.551	0.644	1.253×10^{-6}
SD	0.032	0.033	1.679	5.103	0.523	2.121	0.785	2.273×10^{-5}
Min	0.014	0.014	0.051	1.888	−3.485	−17.167	0.000	0.000×10^{0}
25%	0.051	0.023	0.149	8.249	0.298	-0.614	0.137	2.238×10^{-7}
75%	0.093	0.046	2.315	14.135	0.881	1.198	0.750	8.197×10^{-7}
Max	0.178	0.244	6.875	37.333	2.820	14.175	8.850	2.051×10^{-3}

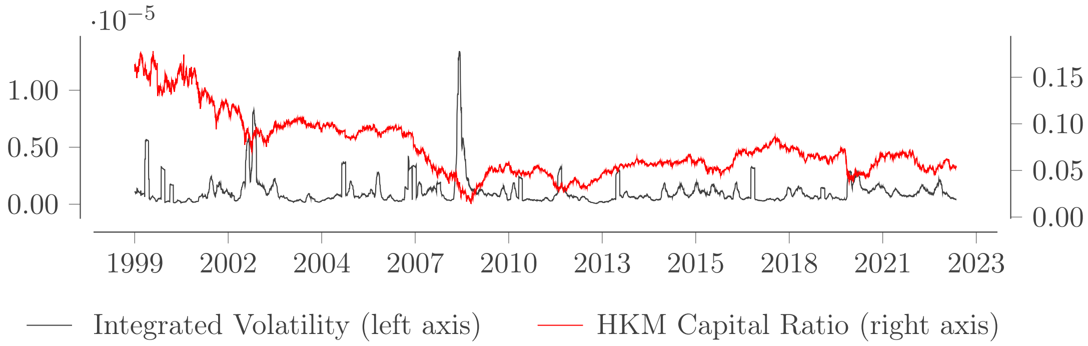
Notes. The table shows the mean, standard deviation (SD), minimum, 25% and 75% percentile, and maximum values for daily frequency control variables used in equations (20) and 21. The variables include (1) HKM, intermediary capital ratio, (2) $EMBI$, emerging Markets Bond Index Plus for Brazil from JP Morgan, (3)i^{US} one-day US Libor rate, (4) i^{BR} one-day Brazil inter-bank rate, (5) i_s^{US}, one-year minus one-day US Libor rate spread, (6) i_s^{BR} one-year minus one-day Brazil inter-bank rate spread, (7) INT^{TOL}, the total amount of interventions in USD (of all instruments), and (8) S^{VOL}, spot market volatility. Values in (3)–(6) are expressed in percentage points. Values in (7) are expressed in billion USD. The sample period runs from 1999-01-22 to 2023-04-27.

B.2 Exchange Rate Variance and Intermediary Constraints

The Gamma-Eta Model in Section 2 assumes a fixed value for the parameter Γ that governs the risk-bearing capacity of the Financier. This assumption departs from the Basic Gamma Model of Gabaix and Maggiori (2015), where Γ depends on the variance of the real exchange rate, $\Gamma = \gamma \operatorname{Var}(e_1)^{\alpha}$, with parameters $\gamma \geq 0$ and $\alpha \geq 0$.

Our empirical proxy for Γ is the intermediary capital ratio of He et al. (2017), which we construct using balance sheet data for a subset of primary dealers that handle emerging-

Figure B.3: Integrated BRL/USD Exchange Rate Volatility and HKM Ratio



Notes. This figure plots the 30-day moving averages of integrated volatility for the BRL/USD spot rate (black) and the HKM capital ratio (red). Integrated volatility is calculated using high frequency spot quotes from Thomson Reuters Tick History. The HKM capital ratio is calculated for a subset of primary dealers that deal with emerging market currencies following He, Kelly and Manela (2017) and Cerutti and Zhou (2024). The sample period runs from 1999-01-22 to 2023-04-27.

market currencies following Cerutti and Zhou (2024). We define this measure in equation (19) and plot it against integrated volatility for the BRL/USD exchange rate in Figure B.3 over a sample period from 1999 to 2023. We compute integrated volatility using high frequency spot quotes from Thomson Reuters Tick History.

We find a weak relationship between the intermediary capital ratio and exchange rate volatility over this period, except perhaps during brief crisis periods, and therefore assume no direct relationship between Γ and $\mathrm{Var}(e)$ in our model.

B.3 Unanticipated Interventions

In Section 4.2 of the main paper, we examine the impact of unanticipated BCB FXI on the BRL/USD exchange rate and currency basis. Figure 7 and Figure 8 in the main paper illustrates our results. In this section of the Online Appendix, we provide a supplemental tabulation of exchange rate effects for each intervention type at discrete post-intervention horizons: 15 minutes, 1 hour, 3 hours, and 7 hours. Table B.4 presents these results. As shown in the table, unanticipated spot sale interventions have the largest and most sustained effects on the BRL/USD exchange rate, while other types of interventions exhibit weaker or transitory impacts.

Table B.4: Exchange Rate Response to Unanticipated BCB Interventions

Panel A: Full Sample

	Spot Sale				Spot Purchase				Traditional Swap				Reverse Swap			
	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours
β_h	−0.332***	−0.663***	−0.752***	−1.951	−0.005	0.012	0.015	0.146	0.002	−0.063	0.001	0.077	0.079**	0.044	0.018	0.062
	(0.096)	(0.241)	(0.276)	(1.193)	(0.013)	(0.024)	(0.025)	(0.089)	(0.104)	(0.131)	(0.147)	(0.276)	(0.032)	(0.056)	(0.078)	(0.111)

Panel B: Tight Intermediary Constraints

	Spot Sale				Spot Purchase				Traditional Swap				Reverse Swap			
	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours
β_h	−0.333***	−0.703***	−0.734**	−1.980	0.025	0.017	0.019	0.063	−0.147***	−0.229***	−0.160	−0.139	0.118**	0.142	0.163*	0.173
	(0.115)	(0.254)	(0.310)	(1.258)	(0.025)	(0.035)	(0.035)	(0.126)	(0.049)	(0.084)	(0.107)	(0.165)	(0.049)	(0.108)	(0.094)	(0.108)

Panel C: Loose Intermediary Constraints

	Spot Sale				Spot Purchase				Traditional Swap				Reverse Swap			
	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours
β_h	−0.187**	−0.106	−0.213	−0.115	−0.033**	−0.007	0.009	0.224**	0.094	0.160	0.185	0.437	0.170***	0.193***	0.100	0.144
	(0.081)	(0.138)	(0.155)	(0.224)	(0.016)	(0.024)	(0.030)	(0.091)	(0.155)	(0.172)	(0.198)	(0.344)	(0.032)	(0.033)	(0.067)	(0.089)

Notes. Panel A shows the exchange rate response to unanticipated BCB FXI at intra-day horizons over the full sample period. Panels B and C show the conditional exchange rate response to unanticipated BCB FXI at intra-day horizons during periods of tight and loose intermediary constraints, respectively. The regressions are specified in equations (20) and (21) in the main paper. White heteroscedasticity-robust standard errors are reported in parentheses. Results are reported in percentage points. Stars *, **, and *** denote 10%, 5%, and 1% significance levels, respectively. The sample period is from 1999-01-22 to 2023-04-27.

B.4 Alternative Measures of Intermediary Constraints

In Section 4.4 of the main paper, we use the *intermediary capital ratio* from He et al. (2017) as an empirical proxy for Γ. A potential concern with this measure is that it may comove with macroeconomic conditions and thus reflect a structural break in our sample, as dealer capital levels tend to be lower after 2008. This could lead to biased estimates of the impact of foreign exchange interventions, as the HKM measure might capture broader post-crisis trends rather than contemporaneous liquidity constraints.

To address this issue, we perform two robustness checks by using intermediaries' positions in foreign exchange markets and CIP violations as alternative proxies for intermediary constraints.

Intermediaries' Positions in Foreign Exchange Markets. Using intermediaries' positions in spot, forward, and futures contracts from the Treasury Foreign Currency (TFC) Reporting,[1] we construct the primary dealers' adjusted foreign exchange position.

In this context, *buying contracts* refers to purchasing non-USD currencies (i.e., selling USD), while *selling contracts* refers to selling non-USD currencies (i.e., buying USD). Dealers are unconstrained when their net contracts purchased are close to zero, as this allows them to maintain inventory positions without accumulating significant exposures. Conversely, positive exposures indicate a net USD intermediation position, reflecting balance sheet constraints faced by dealers. Therefore, the net USD intermediation of dealers relative to the total volume of contracts serves as an aggregate measure of constraints on dealer inventory.

The adjusted foreign exchange position, denoted $FXP_t^{adjusted}$, is defined as

$$FXP_t^{adjusted} = \frac{\sum_i (\text{contracts purchased}_{it} - \text{contracts sold}_{it})}{\sum_i (\text{contracts purchased}_{it} + \text{contracts sold}_{it})}, \tag{37}$$

[1] TFC data collect notional amounts of foreign exchange contracts outstanding, including spot, forward, and futures contracts. The report separates amounts into contracts purchased and contracts sold. Both sides of a foreign exchange transaction are reported. TFC only covers positions in major currencies (CAD, CHF, USD, EUR, GBP, and JPY), which helps mitigate potential endogeneity concerns that could arise from including BRL.

where i indexes individual intermediaries.

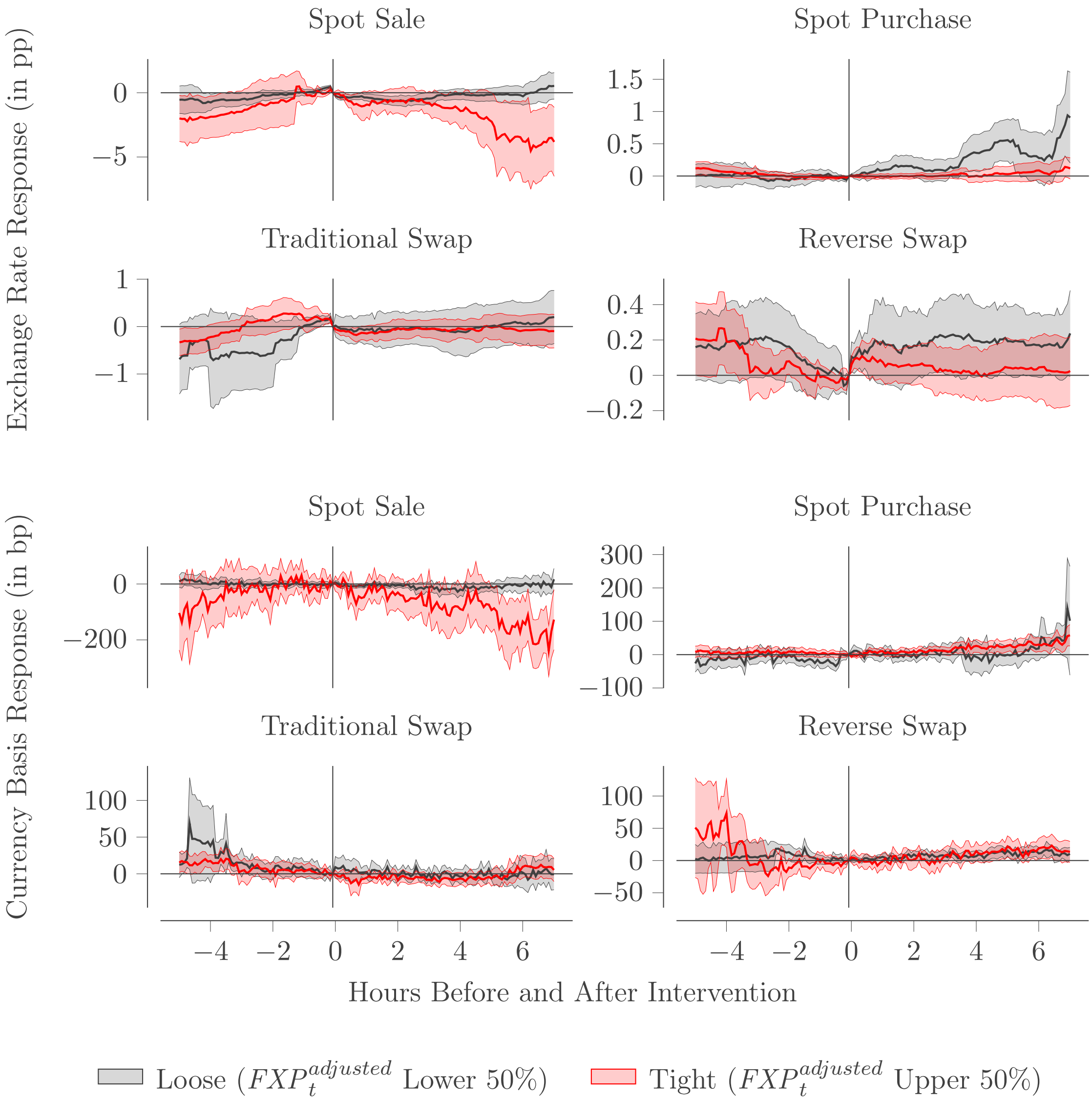
The results in Figure B.4 indicate that our analysis remains robust when $FXP_t^{adjusted}$ is used as the empirical measure of Γ. The upper group of plots in Figure B.4 shows the conditional response of the log spot rate to unanticipated BCB intervention. Similar to the results in the main paper, we find that during periods of tight intermediary constraints, spot USD sales (i.e., buying BRL) can statistically significantly appreciate the BRL against the USD by approximately 4 percentage points over a 7-hour horizon per USD 1 billion sold.

The lower group of plots in Figure B.4 shows the conditional response of the cross-currency basis to unanticipated BCB intervention. We find that the impact of spot USD sales on the cross-currency basis is amplified during periods of tight intermediary constraints, leading to an approximate 200 basis point reduction in CIP violations.

CIP Violations. CIP violations provide a market-based measure of USD liquidity pressures, reflecting the premium required to swap BRL into USD. Unlike intermediary capital ratios, CIP violations are less likely to be influenced by structural shifts and instead offer a more dynamic measure of short-term liquidity constraints faced by intermediaries.

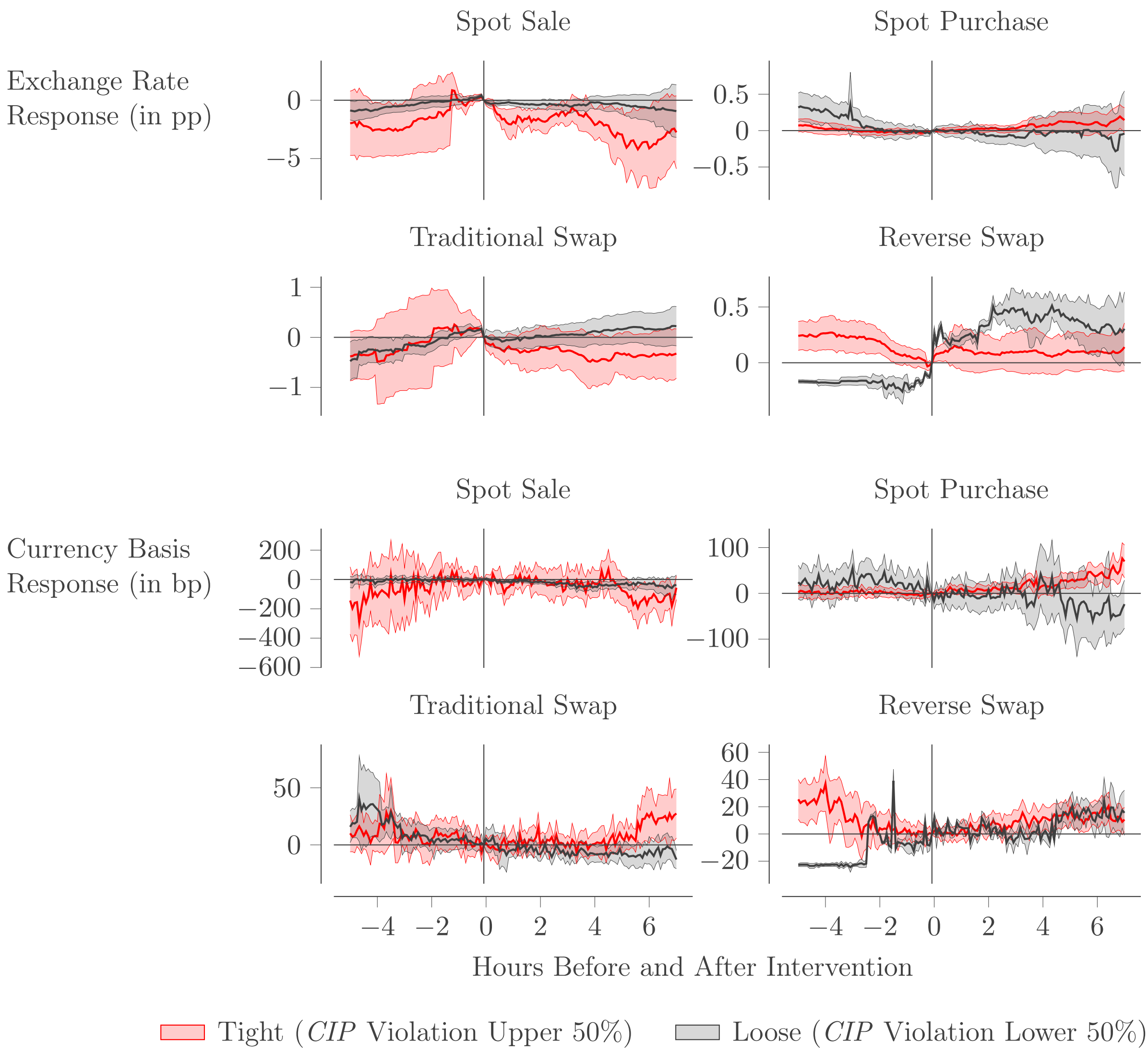
The results in Figure B.5 indicate that our analysis remains robust when CIP violations, measured by the BRL/USD currency basis, are used as the empirical measure of Γ. Figure B.5 shows that spot USD sales (buying BRL) and traditional swap interventions have more pronounced effects on BRL appreciation and the narrowing of the cross-currency basis during periods of above-median CIP violations. These findings align with those obtained using the HKM measure, reinforcing our conclusion that foreign exchange interventions are more effective when intermediaries face tighter constraints.

Figure B.4: Conditional Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions



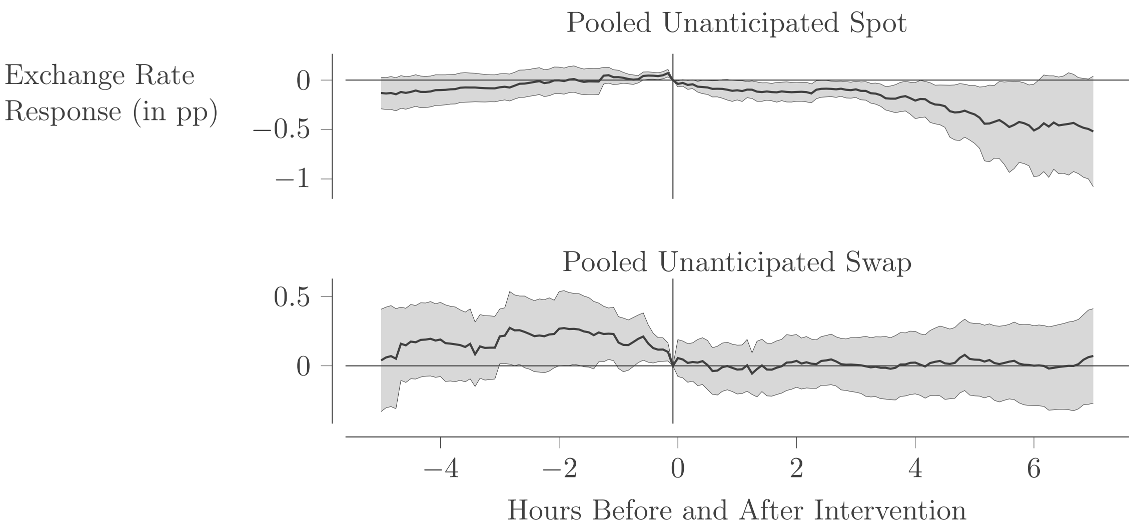
Notes. The upper group of figures show BRL/USD exchange rate responses to unanticipated BCB interventions in percentage points (pp), while the lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions in basis points (bp). Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Each figure plots responses conditional on the state of FX dealers' balance sheet constraints on the intervention day. We measure balance sheet constraints by constructing the primary dealers' adjusted foreign exchange position $FXP_t^{adjusted}$ in equation (37). Red indicates tight constraints, defined as periods with $FXP_t^{adjusted}$ in the upper 50% of values in our sample. Gray indicates loose constraints, defined as periods with $FXP_t^{adjusted}$ in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

Figure B.5: Conditional Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions: CIP Violations



Notes. The upper group figures shows BRL/USD exchange rate responses to unanticipated BCB interventions in percentage points (pp). The lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions in basis points (bp). Each group shows responses to spot sale, spot purchase, traditional swap, and reverse swap interventions. The figures plot responses conditional on the level of CIP violation on intervention day. Red indicates tight-constraint periods with CIP violations in the upper 50% of values, while gray indicates loose-constraint periods with CIP violations in the lower 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

Figure B.6: Exchange Rate Response to Pooled Unanticipated Interventions



Notes. The figure shows the BRL/USD exchange rate response to pooled spot FXI (upper) and pooled swap FXI (bottom), with responses measured in percentage points (pp). In the case of spot interventions, we pool spot sales (with positive sign) and spot purchases (with negative sign) in a single regression. In the case of swap interventions, we pool traditional swaps (with positive sign) and reverse swaps (with negative sign) in a single regression. Unanticipated interventions are measured in USD Billion. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period is from 1999-01-22 to 2023-04-27.

B.5 Pooled Interventions

For our results in Section 4.2 of the main paper, we estimate our baseline specification in (20) separately for interventions in each direction—buy versus sell in the case of spot interventions, and traditional versus reverse in the case of swap interventions. In this section of the Online Appendix, we estimate our baseline specification for pooled interventions. In the case of spot interventions, we pool spot sales (with positive sign) and spot purchases (with negative sign) in a single regression. In the case of swap interventions, we pool traditional swaps (with positive sign) and reverse swaps (with negative sign) in a single regression. Figure B.6 shows the dynamic response of the exchange rate to pooled unanticipated BCB interventions.

The top panel of Figure B.6 illustrates the response to pooled spot interventions, while the bottom panel presents the response to pooled swap interventions. In the pooled analysis, the exchange rate effect of spot FXI diminishes, dropping from our 200 basis point estimate for separate spot sell interventions to 50 basis points for pooled spot buy and sell interventions over a 7-hour horizon. This pooled analysis is valuable for comparing our estimated effect sizes with those from prior literature. Specifically, our 50 basis point pooled estimate falls within the 30-100 basis point range found in previous studies (Kohlscheen and Andrade, 2013; Nedeljkovic and Saborowski, 2019; Barroso, 2019; Santos, 2021).

B.6 Anticipated Interventions

In the main paper, we focus on unanticipated interventions, where we finder stronger effects. In this section of the Online Appendix, we illustrate the effects of anticipated interventions, using the opening of trading as the event time. Figure B.7 shows the exchange rate response to anticipated BCB spot sell interventions (upper panel), as well was traditional and reverse swap interventions (lower panel). Recall that all spot purchase interventions are unanticipated. In all cases, the exchange rate response to anticipated interventions is muted, consistent with Prediction 3 of the Gamma-Eta model.

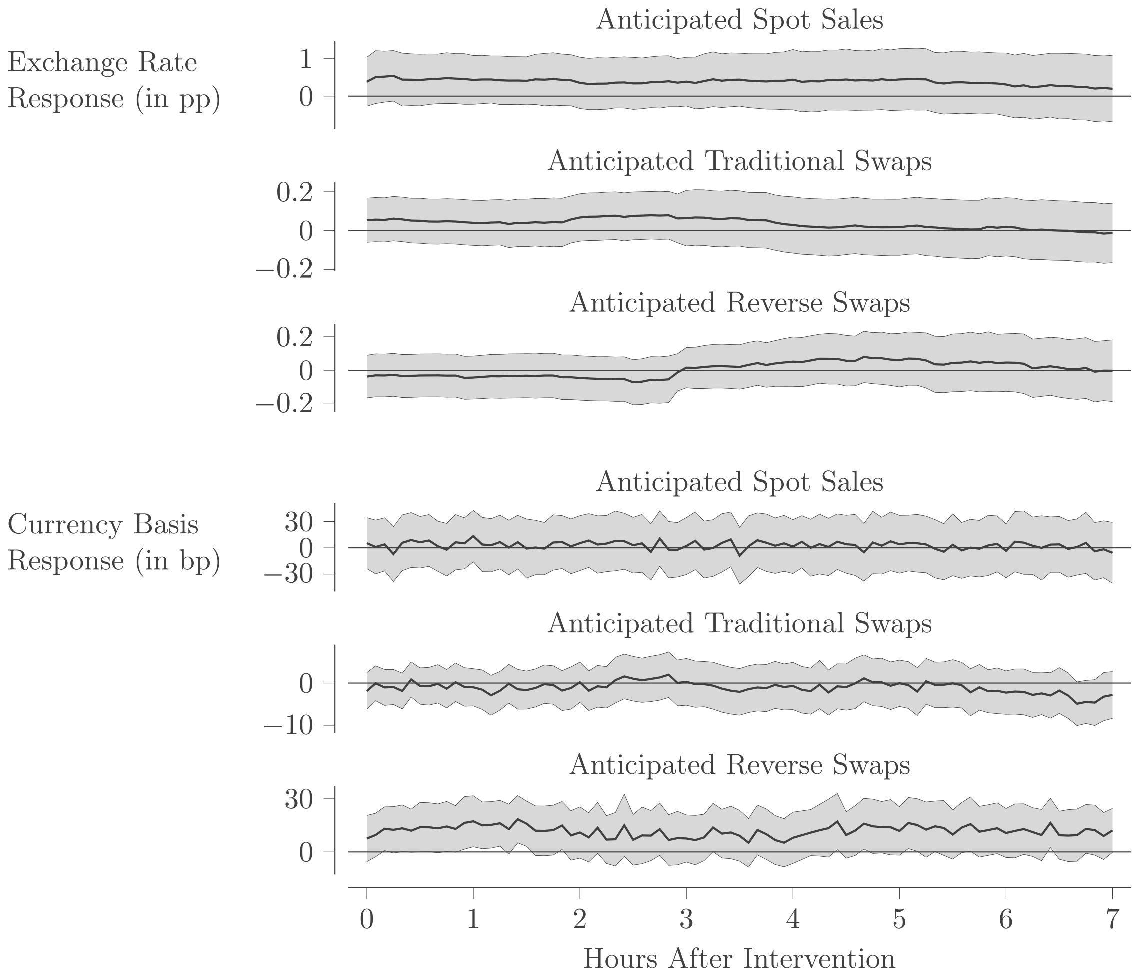
B.7 Forward Premia and Interventions

In this section, we estimate dynamic forward premium responses to unanticipated interventions, providing evidence that links CIP behavior to the relative demand for currency forwards.

Figure B.8 shows the dynamic response of the BRL/USD forward premium to unanticipated FX interventions by the BCB. The figure includes four panels, each representing a different type of intervention: spot sales, spot purchases, traditional swaps, and reverse swaps. The forward premium, calculated as the difference between forward and spot exchange rates, is measured in basis points (bp). The figure shows a positive and significant response in the forward premium to BCB spot sale interventions.

Figure B.9 extends the analysis by conditioning responses on the state of intermediary constraints, as measured by the HKM ratio that we define in equation (19). The figure separates the interventions into two categories: periods with loose constraints (HKM ratio in the upper 50%) and periods with tight constraints (HKM ratio in the lower 50%). The

Figure B.7: Exchange Rate and Currency Basis Responses to Anticipated BCB Interventions



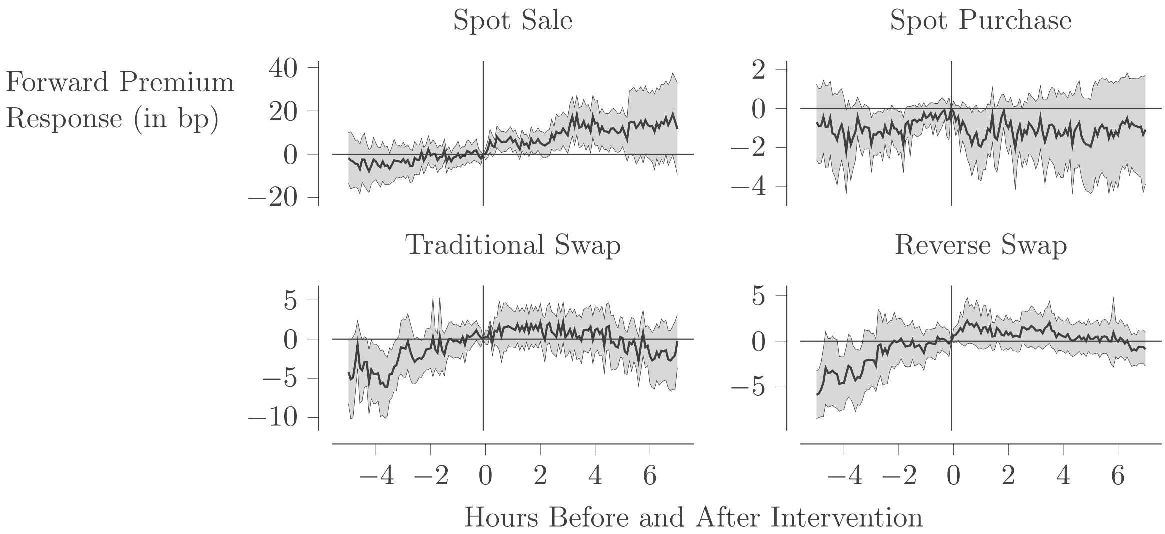
Notes. The upper group of figures show BRL/USD exchange rate responses to anticipated BCB interventions on operational date, measured in percentage points (pp). The lower group shows BRL/USD currency basis responses to anticipated BCB interventions on operational date, measured in basis points (bp). Within each group, the figures show responses to spot sell, traditional swap, and reverse swap interventions. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 1999-01-22 to 2023-04-27.

responses are again presented for spot sales, spot purchases, traditional swaps, and reverse swaps, measured in basis points. This figure shows a slightly more pronounced forward premium response to BCB spot sale interventions.

B.8 Clustered Interventions

In this section, we analyze the differential effects of single and multiple same-day unanticipated BCB interventions on the BRL/USD exchange rate, differentiated by type: spot

Figure B.8: Forward Premium Response to Unanticipated BCB Interventions



Notes. The figures show forward premium responses measured in basis points (bp) to unanticipated BCB spot sales, spot purchase, traditional swap, and reverse swap interventions. The forward premium is defined as the difference between forward and spot BRL/USD rates, measured in basis points. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

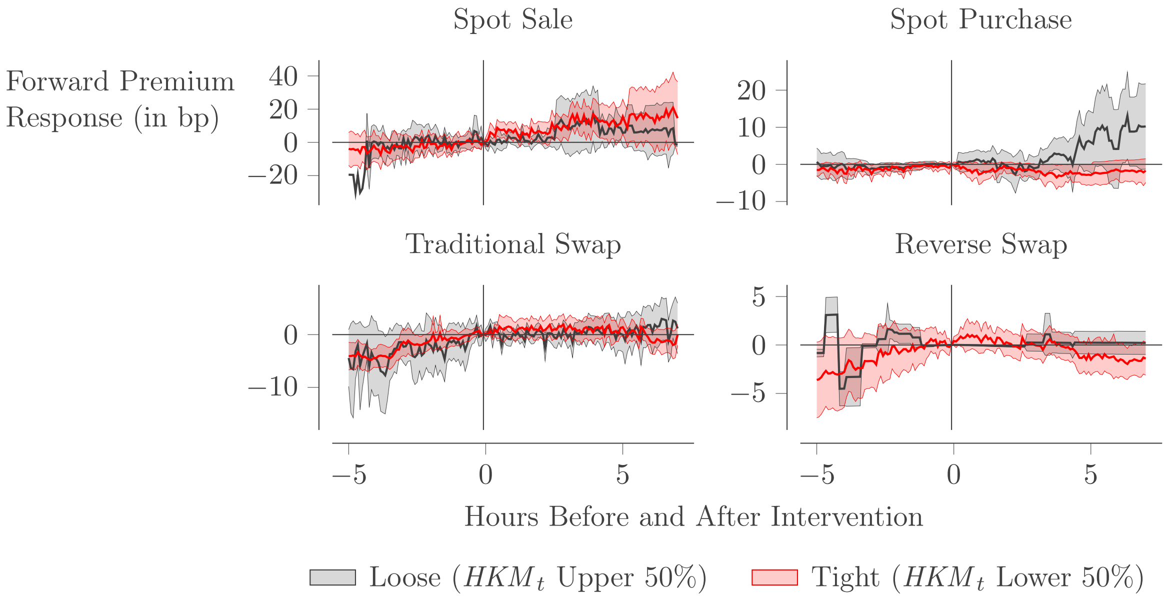
Table B.5: Number of Days with Single or Multiple FXIs

	Spot Sale		Spot Purchase	
	Unanticipated	Anticipated	Unanticipated	Anticipated
$= 1$	252	85	1157	0
$= 2$	54	1	157	0
≥ 3	8	0	4	0
	Traditional Swap		Reverse Swap	
	Unanticipated	Anticipated	Unanticipated	Anticipated
$= 1$	27	67	8	84
$= 2$	48	1055	10	35
≥ 3	48	805	33	175

Notes. This table shows the number of days with single or multiple BCB's FXIs for each type of intervention. For unanticipated interventions, the announcement date is equal to the operation date. For anticipated interventions, the announcement date precedes the operation date. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

sales, spot purchases, traditional swaps, and reverse swaps. The analysis shows how the market responds to isolated interventions versus clusters of interventions occurring within the same day.

Figure B.9: Conditional Forward Premium Response to Unanticipated BCB Interventions

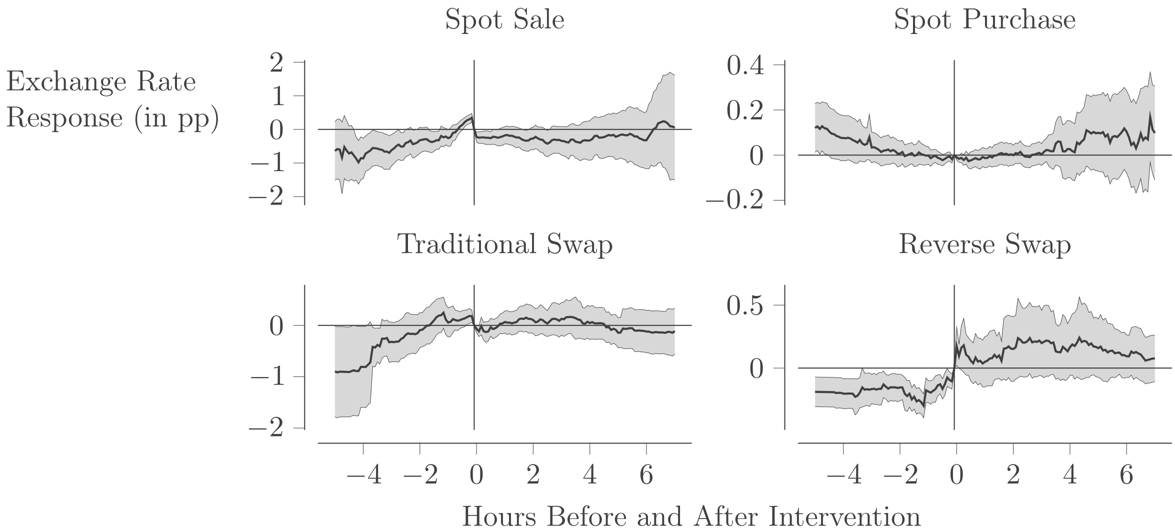


Notes. The figures show BRL/USD 1-month forward premium rate responses in basis points (bp) to unanticipated BCB interventions. The figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Each figure plots responses conditional on the state of FX dealers' balance sheet constraints on the intervention day. We measure balance sheet constraints by constructing the intermediary capital ratio HKM_t of He et al. (2017) for the FX dealer banks listed in Cerutti and Zhou (2024). Red indicates tight constraints, defined as periods with HKM_t in the lower 50% of values in our sample. Gray indicates loose constraints, defined as periods with HKM_t in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

Table B.5 shows the distribution of days with single or multiple FXI by type. For spot interventions, the upper panel indicates that days with single interventions are most common, though days with multiple interventions still occur frequently in our sample. In contrast, the lower panel indicates that days with multiple interventions are most common for swaps. This table motivates our analysis of exchange rate responses to FXI, conditional on the number of same-day interventions.

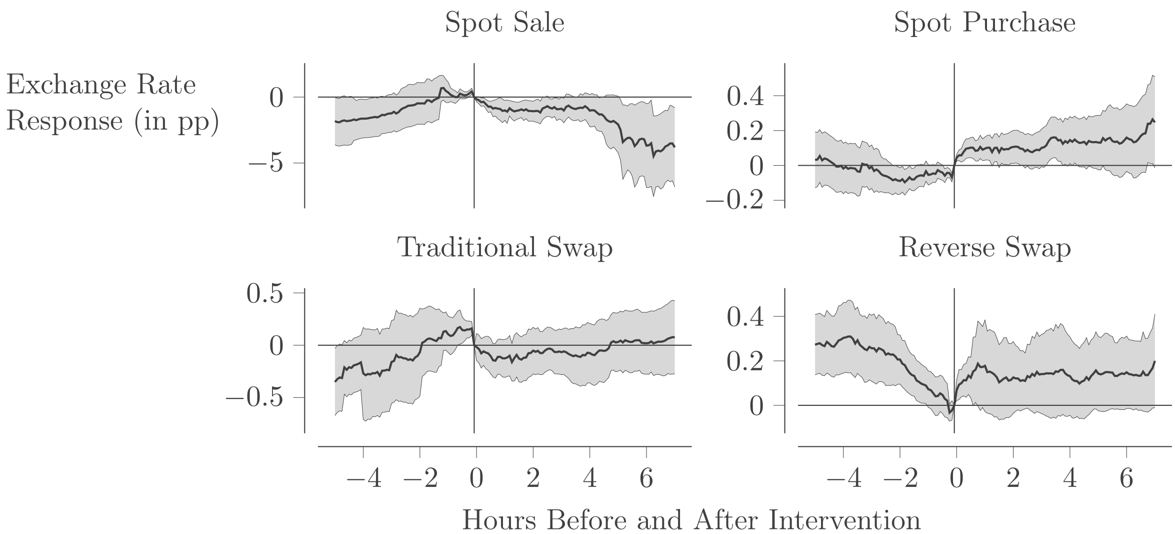
Figure B.10 focuses on days with a single unanticipated intervention, showing somewhat muted exchange rate responses for spot sales, spot purchases, and traditional swaps. Figure B.11 focuses on days with multiple unanticipated interventions, showing stronger and more significant exchange rate responses across these intervention types. Finally, Figure B.12 contrasts the effects of the first intervention of the day with subsequent interventions, showing that subsequent interventions generally produce stronger effects.

Figure B.10: Exchange Rate Response on Days with One Unanticipated Intervention



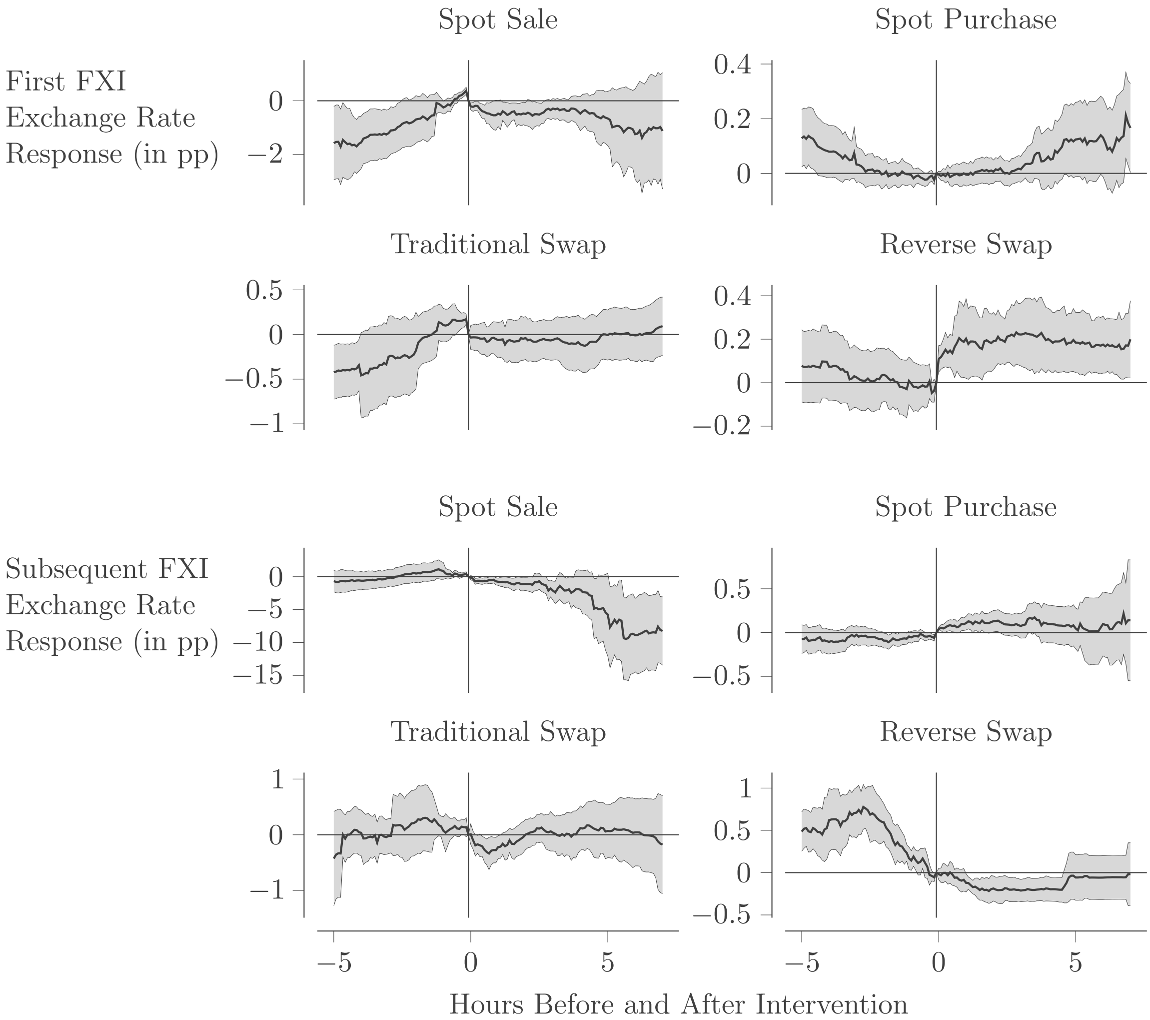
Notes. The figure shows the dynamic response of the BRL/USD exchange rate to BCB spot sale (top left), spot purchase (top right), traditional swap (bottom left), and reverse swap (bottom right) FXI on days with exactly one BCB intervention. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White heteroscedasticity-robust standard errors. The sample is from 2003-11-24 to 2023-04-27.

Figure B.11: Exchange Rate Response on Days with Multiple Unanticipated Interventions



Notes. The figure shows the dynamic response of the BRL/USD exchange rate to BCB spot sale (top left), spot purchase (top right), traditional swap (bottom left), and reverse swap (bottom right) FXI on days with more than one BCB intervention. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White heteroscedasticity-robust standard errors. The sample is from 2003-11-24 to 2023-04-27.

Figure B.12: Exchange Rate Response to First and Subsequent Unanticipated Interventions

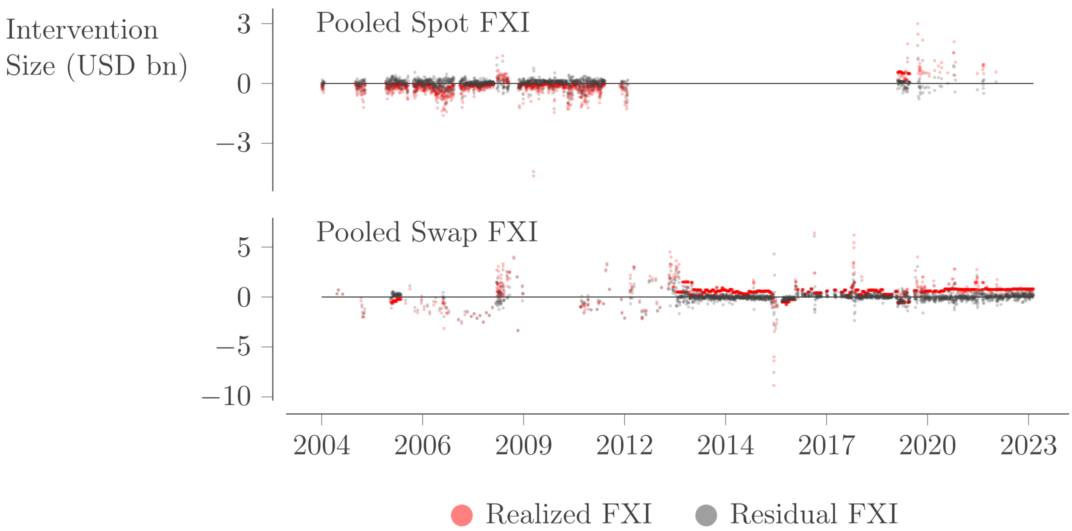


Notes. The upper group of figures shows the BRL/USD exchange rate responses to the first unanticipated BCB intervention on days with multiple unanticipated interventions, measured in percentage points (pp). The lower group of figures shows the BRL/USD exchange rate responses to subsequent unanticipated same-day BCB interventions. Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 1999-01-22 to 2023-04-27.

B.9 Residual Intervention Measure

In Section 4.5 of the main paper, we examine the signaling channel of FXI and in part of our examination we use daily-frequency data to estimate our baseline specification in (20) over a six-month horizon. For this lower-frequency, longer-horizon analysis, we construct a residual measure of FXI in order to mitigate policy endogeneity concerns. In this section of the Online Appendix, we describe our procedure for obtaining the plausibly-exogenous

Figure B.13: Exogenous Residual Component of FXI



Notes. The figure shows realized values of FXI in red, along with the plausibly-exogenous residual component of FXI in grey, for BCB FXI aggregated at a daily frequency over the period 1999 to 2023. The upper panel shows realized pooled spot intervention amounts and residuals, and the lower panel shows pooled swap intervention amounts and residuals. The linear FXI prediction model we use to estimate the residual component of FXI is defined in equation (38).

residual component of FXI.

Specifically, we regress the intervention amount FXI_t on a set of control variables, interpreting the residual from this regression as the exogenous component of FXI. In the case of spot interventions, we pool buy and sell interventions for the regression, where spot sell interventions receive a positive sign and spot buy interventions receive a negative sign. In the case of swap interventions, we pool traditional and reverse swap interventions for the regression, where traditional swap interventions receive a positive sign and reverse swap interventions receive a negative sign.

Our specification is given by

$$FXI_t = \alpha + \beta z_{t-1} + \epsilon_t \, , \tag{38}$$

for all dates on which FXI is non-zero, i.e. for all t such that $FXI_t \neq 0$, where FXI_t denotes the dollar amount of FXI in USD billion. The vector of control variables z_{t-1} includes lagged Brazilian interest rate expectations, the BRL/USD spot rate, the HKM intermediary capital ratio, sovereign default risk, exchange rate volatility, U.S. and Brazilian interest rates, monetary policy announcements, US recessions, forward premia, and FX interbank trading volume. We plot the realized FXI values and residuals in Figure B.13.

We interpret the residual from this regression as the plausibly-exogenous component of FXI, and we use the residual in a second stage to estimate our baseline local projections specification in (20). Our procedure is similar to the procedure that Rodnyansky et al. (2023) use to identify an exogenous component of FXI.

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