Who Carries?*

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Abstract

In representative agent models, consumption growth risk explains currency carry trade returns only when risk aversion is very high. Yet, a highly risk-averse agent would not hold a carry trade portfolio. Heterogeneity helps by allowing a risk-tolerant minority of agents to hold carry trade portfolios while a risk-intolerant majority does not. We show that with heterogeneous risk aversion, standard models of international macroeconomics can produce carry traders in economies with domestic bias in aggregate portfolios and low aggregate portfolio returns, as observed in Germany, Japan, and the United States, together holding half of global debt.

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1 Introduction

The carry trade has two parts: the return and the portfolio position. Most research on the carry trade in foreign exchange markets seeks to rationalize the return but not the position. A satisfactory theory should rationalize both. In particular, a theory of the carry trade must grapple with the empirical observation that aggregate portfolio positions rarely resemble aggregate carry trades. In representative-agent economies without aggregate carry trades, the representative agent cannot be a carry trader.

This tension is evident in existing research. Consider the framework of Lustig and Verdelhan (2007), where carry trade returns compensate an agent for holding high interest-rate currencies that depreciate when consumption growth is low. To match carry trade returns for the United States, they assume the agent is highly risk averse. While their framework does not pin down portfolio positions, standard extensions would imply domestic portfolio bias for the highly risk averse agent. Domestic portfolio bias would fit the data for the United States, but would look nothing like a carry trade, because interest rates in the United States have been low in recent decades. The same is true of Germany and Japan, as we report in Table 1. For these three economies, which together hold half of global debt, and others like them, we ask: who carries?

Heterogeneity offers a compelling answer. By fixing aggregate risk aversion at a high level while allowing for within-country variation across agents, it should be possible to generate plausible carry trade returns for a minority of risk-tolerant agents in an economy with domestic portfolio bias and low portfolio returns in the aggregate. Indeed, we prove sufficient conditions for exactly this result in a standard two-country dynamic stochastic general equilibrium model with incomplete markets that we augment with heterogeneity in risk aversion. The benefits of this approach are substantial.

First, heterogeneity bridges the gap between the finance literature on currency carry trades and the international macroeconomics literature on domestic bias in aggregate portfolio holdings. The finance literature typically employs representative-agent models to study carry trade returns. However, as discussed, the representative agent is not a carry trader in countries earning low returns on portfolios with domestic bias in aggregate—an empirically important case. Conversely, the international macroeconomics literature typically employs representative-agent models to study domestic bias in aggregate portfolio holdings, placing little emphasis on relative returns across currencies. Our heterogeneous-agent framework can simultaneously capture *individual carry trade positions* and *domestic bias in aggregate positions* in economies with low aggregate portfolio returns.

Second, the heterogeneity that drives the carry trade in our model also drives a wedge between the aggregate coefficient of risk aversion and the aggregate elasticity of intertemporal substitution. This aggregation wedge arises even for isoelastic utility functions with constant relative risk aversion. It transforms these preferences into a parsimonious and tractable alternative to the recursive preferences traditionally employed to separate risk aversion from intertemporal substitution. The aggregation wedge affects the responsiveness of macroeconomic variables to shocks, so the same heterogeneity that gives rise to individual carry traders also affects macroeconomic dynamics.

Third, the domestic bias in aggregate portfolios that our model generates raises the responsiveness of carry trade returns to shocks. In the model, a portfolio with domestic bias amounts to a hedge against low consumption growth. If consumption growth falls, hedged portfolio values rise. When a majority of agents are hedged, this initial rise produces a positive feedback effect that raises hedged portfolio values further. The feedback effect improves the hedging properties of portfolios with domestic bias, but worsens the anti-hedging properties of portfolios with international bias, which represent carry trade portfolios in the model. Thus, domestic bias in aggregate portfolios amplifies the risk of the carry trade, and carry traders earn higher returns.

We acknowledge at the outset one limitation of our approach: for asset pricing, the aggregate risk aversion that heterogeneous-agent models require exceeds the already high level that representative-agent models require. Consider Lustig and Verdelhan's model cited above, and reinterpret their pricing equation (3) for a continuum of heterogeneous agents. For an individual agent, the equation can be written as

$$\frac{1}{\gamma-1}\,\mathrm{E}\Big[R_{i-j}\Big] \approx b_1'\,\mathrm{Cov}\Big(\Delta c(\gamma), R_{i-j}\Big) + b_2'\,\mathrm{Cov}\Big(\Delta d(\gamma), R_{i-j}\Big),$$

where γ denotes the agent's coefficient of relative risk aversion, R_{i-j} denotes the return differential $R_i - R_j$, with R_i and R_j denoting USD returns on assets denominated in currencies i and j, and $\Delta c(\gamma)$ and $\Delta d(\gamma)$ denoting the agent's non-durable and durable consumption growth, respectively.[1] We define $b_1' \equiv b_1(\gamma)/(\gamma-1)$ and $b_2' \equiv b_2(\gamma)/(\gamma-1)$ to be independent of γ, where $b_1(\gamma)$ and $b_2(\gamma)$ are agent-specific reinterpretations of b_1 and b_2 defined in Lustig and Verdelhan's equation (4). Assuming a density function $f(\gamma)$ with support over $(1, \gamma_m]$ to describe the distribution of risk aversion across agents, we integrate to recover the aggregate pricing equation,

$$\mathrm{E}\Big[R_{i-j}\Big] \approx \frac{b_1}{\omega}\,\mathrm{Cov}\Big(\Delta c, R_{i-j}\Big) + \frac{b_2}{\omega}\,\mathrm{Cov}\Big(\Delta d, R_{i-j}\Big),$$

where b_1 and b_2 are the original coefficients in Lustig and Verdelhan (2007), but where an aggregation wedge $\omega > 1$ now appears as a consequence of Jensen's inequality.[2] With the wedge, the aggregate risk aversion embedded in b_1 and b_2 must be larger to explain

[1]We omit time subscripts and simplify Lustig and Verdelhan's equation (3) by assuming that assets R_i and R_j covary equally with the return on each agent's wealth. Our arguments hold without this simplification.

[2]With $\gamma > 1$, f non-degenerate, and $\mathrm{E}_\gamma\big[g(\gamma)\big] = \int_1^{\gamma_m} g(\gamma)f(\gamma)\,\mathrm{d}\gamma$, Jensen's inequality implies $\mathrm{E}_\gamma\big[1/(\gamma-1)\big] > 1/(\mathrm{E}_\gamma[\gamma]-1)$. A wedge $\omega > 1$ equates the two sides, $\mathrm{E}_\gamma\big[1/(\gamma-1)\big] = \omega/(\mathrm{E}_\gamma[\gamma]-1)$.

Table 1 – Domestic Currency Bias and Returns on Aggregate Debt Portfolios

	Group Share of Global Holdings	Country-Level Weighted Average (in %)		
		Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return
Germany, Japan, United States	54.0	91.4	1.2	−0.3
All Other Countries	46.0	73.0	3.2	0.7

Notes. The table reports estimated domestic currency bias and returns on debt portfolios held by Germany, Japan, and the United States versus 181 other countries. Portfolio returns and domestic currency bias are estimated from OECD and IMF interest rates, exchange rates, and CPI inflation, combined with portfolio weights estimated from IMF and BIS debt holdings and issuance data over the period 2001–2021. Shares of global holdings are calculated for each group then averaged over years; all other values are calculated for each country then averaged over countries and years, using each country's debt holdings as weights. We exclude outlier interest rates and exchange rate returns exceeding three standard deviations from annual means. Appendices A.2 and A.3 describe our methodology, with robustness checks in Table A3.

carry trade returns. Although heterogeneous-agent models raise the level of risk aversion required to explain carry trade returns, the level was already implausibly high in the representative-agent case, and the increase can be made arbitrarily small by skewing the distribution of risk aversion. Therefore, we consider the additional cost of heterogeneity to be marginal relative to the benefits it brings.

We empirically motivate our study using the International Monetary Fund's Coordinated Portfolio Investment Survey to show that carry traders are outweighed by other investors in debt markets. First, for the eight countries that report them, bilateral aggregate *gross* short positions rarely exceed corresponding bilateral gross long positions. Second, across 86 countries reporting bilateral aggregate *net* positions, less than 0.2% of reported positions are net negative, further suggesting long positions outweigh short positions, even in low interest-rate currencies. Third, we estimate domestic-currency bias, revealing that low interest-rate countries like Germany, Japan, and the United States exhibit stronger domestic-currency bias than many high interest-rate countries, as documented in Table 1. Taken together, this evidence suggests that investors who prefer domestic-currency assets outweigh carry traders in debt markets. While carry trades may be more prevalent in derivatives markets, a growing consensus indicates that hedging, not speculation, dominates even these markets, as we review below.

We rationalize these empirical patterns in a dynamic stochastic general equilibrium model with an infinite horizon, incomplete markets, two countries, and a continuum of agents in each country with heterogeneous risk aversion. In the model, individual coefficients of relative risk aversion equal the inverse of individual elasticities of intertemporal substitution, but this inverse relationship breaks when the individual parameters are aggregated. If aggregate risk aversion exceeds a low threshold, the aggregate portfolio displays domestic bias. While the majority of individual agents endogenously hold portfolios with domestic bias, a minority of agents hold carry trade portfolios, consistent with the evidence.

Importantly, we present a novel method to solve for both individual and aggregate portfolio holdings, extending existing methods in international macroeconomics to a new class of heterogeneous-agent models. Our model admits approximate closed-form solutions for individual and aggregate consumption, real wealth, and real portfolio holdings, as well as for the real exchange rate and the carry trade real return. Conveniently, heterogeneity vanishes in the model's non-stochastic steady state, and the model pins down a degenerate cross-sectional distribution of zero steady-state real wealth. Around this steady state, the model generates a stationary, non-degenerate cross-sectional distribution of real wealth and consumption. Throughout the model, we prioritize tractability and simplicity in order to emphasize the mechanisms at work and highlight the key insights. For this reason, we abstract from many complexities that enrich medium-scale models of international macroeconomics.

The remainder of the paper is organized as follows. In Section 2, we discuss related literature. In Section 3, we make an empirical case for heterogeneity and domestic-currency bias in portfolio positions by examining country-level debt holdings. In Section 4, we construct our two-country dynamic stochastic general equilibrium model with heterogeneity in risk aversion. Section 5 concludes.

2 Related Literature

This paper bridges existing research on currency carry trades and domestic bias in portfolio holdings, using preference heterogeneity to develop a unified framework that explains both investment patterns.

In foreign exchange markets, carry trades exploit interest rate differentials across currencies. These trades violate uncovered interest parity in predictable ways, as documented by Hansen and Hodrick (1980), Fama (1984), and Engel (1984). Most explanations for these violations fall into one of three categories: financial frictions (Burnside et al., 2006; Brunnermeier et al., 2009; Gabaix and Maggiori, 2015), rare disasters (Barro, 2006; Burnside et al., 2011; Farhi and Gabaix, 2016), and risk-based factors (Lustig and Verdelhan, 2007; Lustig et al., 2011; Menkhoff et al., 2012; Lettau et al., 2014).

The consumption-based Capital Asset Pricing Model (CCAPM), adapted to carry trades by Lustig and Verdelhan (2007), argues that carry trade returns compensate investors for exposure to consumption growth risk. Despite some debate (Burnside, 2011; Lustig and Verdelhan, 2011), CCAPM remains influential in risk-based explanations.

Within this tradition, we are closest to Jylhä and Suominen (2011) and Filipe et al. (2023), who develop a two-country model with endogenous carry traders using two types of myopic agent—one restricted to domestic markets, the other unrestricted. Relatedly, Park (2022) uses heterogeneous beliefs in a two-country, two-period model with three agent types, to show that pessimistic beliefs can increase carry trade returns. In the financial frictions tradition, Richers (2019) studies portfolio positions in a two-period model with a global investor and local agents, and agency costs that produce persistent return differentials across countries. Like these papers, we also model carry traders in an economy with heterogeneous agents.

Unlike these papers, we model a continuum of unrestricted agents with heterogeneous risk aversion who optimize over an infinite horizon. Our framework incorporates richer heterogeneity into a standard international macroeconomic model, uniquely showing how carry traders emerge endogenously in economies where most agents prefer low-return portfolios with domestic bias. Empirically, our findings differ from Richers, who reports higher returns on aggregate debt portfolios with domestic bias, based on a smaller sample of countries excluding the Euro area and the United States. In contrast, we find that many developed countries, including Germany, Japan, and the United States, earn lower returns on aggregate debt portfolios with strong domestic bias.

The profitability of the carry trade has been extensively studied, but evidence on actual carry trade positions is more limited. Galati et al. (2007) and Gagnon and Chaboud (2007) find modest evidence of carry trades primarily in FX derivatives markets. Curcuru et al. (2011) and Fong (2013) also document limited evidence of carry trades, even during times of reportedly high activity. Recent BIS evidence identifies hedge funds and global macro investors as main participants in carry trade strategies, primarily in FX derivatives markets (Bank for International Settlements, 2024). Using UK transaction-level data, Hacioğlu-Hoke et al. (2024) confirm hedge funds as important carry traders, but their positions represent only about 3% of net FX derivatives exposures. Specific carry trade channels have been observed in particular contexts—household foreign currency borrowing in Austria (Beer et al., 2010), European bank behavior during the sovereign debt crisis (Acharya and Steffen, 2015), and emerging market firms issuing USD bonds (Bruno and Shin, 2017)—but these specialized cases do not suggest widespread dominance of carry trade strategies.

In contrast, a growing literature finds that hedging, not carry trade, drives much of the currency derivatives market. Allayannis and Ofek (2001) and Allayannis et al. (2001) find that large U.S. firms use currency derivatives to reduce exchange rate exposure, enhancing firm value. Bartram et al. (2010) and Bartram (2019) find that non-financial firms globally use currency derivatives to reduce exchange rate exposure. Bräuer and Hau (2022) show that institutional investor hedging explains significant exchange rate variation. Sialm and Zhu (2024) find that most U.S. international fixed income mutual funds hedge with currency forwards. Hacioğlu-Hoke et al. (2024) show that pension and investment funds, insurers, non-financial corporations, and non-dealer banks, unlike hedge funds, use currency derivatives primarily for hedging. Alfaro et al. (2024) survey evidence from transaction-level datasets on currency hedging with financial derivatives. Liao and Zhang (2025) find that hedging demands rise in periods of financial distress, affecting exchange rates.

In bond markets, domestic bias dominates. Lane (2005) finds strong regional bias in Euro-area bonds after the Euro's introduction, anticipating a later literature on domestic-*currency* bias. Fidora et al. (2007) and Coeurdacier and Rey (2013) find that domestic bias persisted more strongly in bonds than in equities over decades of financial globalization. Coeurdacier and Gourinchas (2016) show that bonds help hedge real exchange rate risk in portfolios with equities. Burger et al. (2018) find that investors strongly prefer domestic-currency bonds, with U.S. investors over-weighting USD bonds from foreign issuers. Recent work by Maggiori et al. (2020) and Florez-Orrego et al. (2023) using micro data shows that global investors prefer domestic-currency bonds, with USD bonds as an exception. Faia et al. (2022) find domestic bias in Euro-area corporate bond holdings for insurers and pension funds but not mutual funds. Burietz and Ureche-Rangau (2020) find domestic bias in Euro-area syndicated lending.

These studies collectively suggest that while carry traders exist in both currency derivatives markets and specialized debt markets, they do not represent the dominant market participants. Hedging appears to be more important than speculation for many non-financial and financial firms. These findings reinforce our thesis that carry traders, while present, are outweighed by investors with non-speculative motives. We reconcile these findings with the carry trade literature using a heterogeneous-agent framework.

Our framework builds on influential methodological advances in solving for international portfolios in open-economy models with incomplete markets. Guu and Judd (2001), Devereux and Sutherland (2010, 2011), Tille and van Wincoop (2010), and Evans and Hnatkovska (2012) develop methods to solve open-economy models with portfolio choice under incomplete markets using perturbation methods. More recently, Saucet (2022) develops a global solution for portfolio holdings in an open-economy with heterogeneity across but not within countries. Our technical contribution extends the Devereux and Sutherland (2011) perturbation method to allow for within-country heterogeneity in risk preferences for a continuum of agents (in addition to heterogeneity across countries). We provide approximate closed-form solutions for the distinct portfolio of each individual agent, and for the aggregate economy. While we solve only for steady-state holdings, this limitation can be addressed in future work using third-order approximations.

Our work relates to a macroeconomics literature on preference heterogeneity that until recently followed the Bewley-Ayagari-Hugget tradition, where agents differ only in their history of idiosyncratic shocks. This was true of earlier HANK models (see Kaplan et al.

2018), where heterogeneity in the marginal propensity to consume (MPC) affects monetary policy transmission. Recently, Aguiar et al. (2020), Calvet et al. (2021) and Gelman (2021) have shown that preference heterogeneity rather than shocks can explain much of the observed heterogeneity in MPCs. Kekre and Lenel build New-Keynesian models with two-type heterogeneity in risk aversion—across households in a closed economy (Kekre and Lenel, 2022) and across countries in an open economy (Kekre and Lenel, 2024). Davis and Van Wincoop (2024) develop an open-economy model featuring within-country heterogeneity in risk aversion and many types; unlike us, they focus on capital flow retrenchment and model portfolio choice over a two-period horizon.

Our work also relates to a finance literature on preference heterogeneity. Kogan and Uppal (2001) study portfolio holdings and asset pricing in a closed economy with heterogeneous risk aversion and incomplete markets, but unlike us they perturb parameters of the utility function around the logarithmic utility case. Guvenen (2009) studies asset prices in a two-agent closed-economy macro model with limited stock market participation and heterogeneity in the intertemporal elasticity of substitution (IES). Garleanu and Panageas (2015) study asset pricing in a closed-economy OLG model with two agents distinguished by their IES and risk aversion, and Panageas (2020) surveys the literature on closed-economy asset pricing with heterogeneity and provides a unifying framework.

This paper offers a unique combination of features that distinguishes it from prior research on preference heterogeneity. While existing papers share individual elements with our approach, none combines: (1) a continuum of agents with heterogeneous risk aversion rather than a limited number of types; (2) agents optimizing over an infinite horizon rather than myopic agents or two-period frameworks; (3) an open-economy setting that captures both within and across-country heterogeneity; and (4) unrestricted agents who can freely participate in both domestic and international markets. This combination allows us to study the effects of heterogeneity on individual and aggregate variables in a benchmark open-economy model, while explaining how domestically biased investors, hand-to-mouth consumers, and carry traders emerge endogenously from the same underlying preference distribution.

3 Empirical Motivation

We begin with the observation that if most individual investors within a country took carry trade positions in debt markets, we would observe negative aggregate positions in debt denominated in low interest-rate funding currencies. In the data, we nearly never observe this. Country-level aggregate portfolio positions, which equal the sum of individual investor positions, are positive for nearly every counterparty country and year. It follows that individual investors with long positions nearly always outweigh individual investors with short positions, even in low interest-rate debt. Furthermore, we find that investors hold domestic and domestic-currency debt disproportionately, especially in low interest-rate countries. In view of this evidence, we argue that carry traders are outweighed by non-carry traders in debt markets.

Our empirical analysis uses aggregate data from the International Monetary Fund's Coordinated Portfolio Investment Survey (CPIS). Section 3.1 examines an unbalanced panel of six countries that report aggregate gross short and aggregate net positions in the debt of 236 issuer countries over an eight-year period. We find little evidence of significant aggregate gross short positions. Section 3.2 expands this analysis to a broader set of 86 countries that report aggregate net rather than gross positions. For this broader set of countries, we find strikingly few cases of negative aggregate net holdings of debt, and this finding does not seem to be driven by misreporting. Section 3.3 documents strong domestic-currency bias in estimated aggregate net holdings of debt for an even broader cross-section of 181 countries. The domestic-currency bias we estimate corroborates the findings of Maggiori et al. (2020), who document a similar bias in granular security-level data.[3]

We acknowledge that CPIS data limits our analysis: we cannot study individual carry trade positions in this data, nor can we study carry trade positions in derivatives markets. But our aim is equally limited: we seek only to establish whether or not carry traders are reasonably representative investors. This limited aim justifies our use of the CPIS data. Debt markets are large, CPIS has broad coverage across countries, and the absence of dominant carry trade patterns here is compelling.

While we find no evidence of widespread carry trade activity in debt markets, we find strong evidence of domestic bias in these markets, especially in developed economies with low interest rates. Overall, we conclude that carry traders should not be viewed as representative because a significant portion of investors pursue non-carry trade strategies, motivating the need for carry-trade models that incorporate investor heterogeneity.

3.1 Aggregate Gross Short Positions by Issuer Country

Eight countries report their aggregate gross short positions in foreign debt separately from their aggregate net positions in CPIS. These annual reports cover up to 236 counterparty countries over an eight-year period. Most reported positions are zero-valued, with non-zero positions reported for only 61 counterparty countries. Among the reporting countries, only the Cayman Islands and Germany report significant positions. Aruba, Belgium, Bulgaria, Cyprus, Estonia, and Lithuania report either zero or negligible positions. While limited

[3]We do not reallocate holdings in tax havens following Coppola et al. (2021), as this reallocation could obscure carry trade activity, which may occur disproportionately in tax havens (Curcuru et al., 2011). Without this reallocation, our estimates of domestic currency bias in Appendix A.3 likely understate the true extent of the bias, because a fraction of holdings in tax havens may represent holdings of domestic assets.

Table 2 – Average Aggregate Gross Short Positions: Cayman Islands and Germany

Holding Country	Average Annual Values			
	Aggregate Gross Short Positions (in USD millions)	Short Ratio (in %)	Count of Aggregate Gross Short Positions	
			Non-Zero	Non-Missing
Cayman Islands	−3 733	6	363	902
Germany	−98	1	333	1 182

Notes. The table summarizes IMF CPIS data on aggregate gross short positions. It shows average positions (USD millions), average short ratios defined in equation (1), and average annual counts of non-zero and non-missing positions, 2015–2023. All values are averaged over counterparties and years, and rounded to the nearest integer.

in scope, this evidence may generalize to other offshore financial centers and developed economies, and suggests that aggregate gross short positions in debt are generally small and rare.

Table 2 summarizes reported aggregate gross short positions in debt for the Cayman Islands and Germany from 2015 to 2023. The positions reported by the Cayman Islands, an offshore financial center, stand out in terms of value. Averaging across counterparty countries and over years, the Cayman Islands report an average position that is over 30 times greater in value than that of Germany, while reporting a roughly equal number of non-zero positions. Even so, the reported gross short positions of the Cayman Islands are relatively small.

To measure their relative size, we define the short ratio as

$$SR_{ij} = \frac{-B_{ij}^{k\,\text{Short}}}{B_{ij}^{k\,\text{Long}} - B_{ij}^{k\,\text{Short}}}, \tag{1}$$

where $B_{ij}^{k\,\text{Short}}$ and $B_{ij}^{k\,\text{Long}}$ denote, respectively, the aggregate gross short and aggregate gross long positions of country i in debt issued by country j and expressed in numéraire currency k (USD in the CPIS data). The short ratio lies between zero and one, and if a country's aggregate gross short position exceeds its aggregate gross long position with a counterparty, the short ratio exceeds one half. As Table 2 shows, the average Cayman short ratio is six times greater than the average German short ratio, but far lower than one half.

Table 3 presents the Cayman and German top five destination countries for aggregate gross short positions, ranked by cumulative value over 2015–2023. Notably, Japan is absent from these top-five lists, despite the attractiveness of Japanese debt as a low-cost funding asset for carry trades. This finding aligns with Gagnon and Chaboud (2007), who report mixed evidence of Yen-funded carry trade positions across Japanese sectors and limited evidence outside Japan. Unlike Gagnon and Chaboud, we exclude balance sheet

Table 3 – Aggregate Gross Short Position Destinations: Cayman Islands and Germany

Cayman Islands' Top Destinations for Gross Short Positions		Germany's Top Destinations for Gross Short Positions	
Issuing Country	Cumulative Gross Short Position (in USD millions)	Issuing Country	Cumulative Gross Short Position (in USD millions)
Germany	−1 106 559	Israel	−18 942
United States	−756 679	France	−17 691
France	−493 238	United Kingdom	−17 335
United Kingdom	−340 584	Italy	−12 082
Italy	−217 794	Spain	−7 980

Notes. The table shows the top five destination countries for Cayman and German aggregate gross short positions, ranked by cumulative value. Cumulative gross short positions are aggregate gross short positions summed by issuing country over the period 2015–2023, reported in USD millions.

liabilities and focus on active long and short positions booked as balance sheet assets. Nevertheless, both our findings and theirs suggest diverse investor motivations beyond carry trade returns, indicating investor heterogeneity.

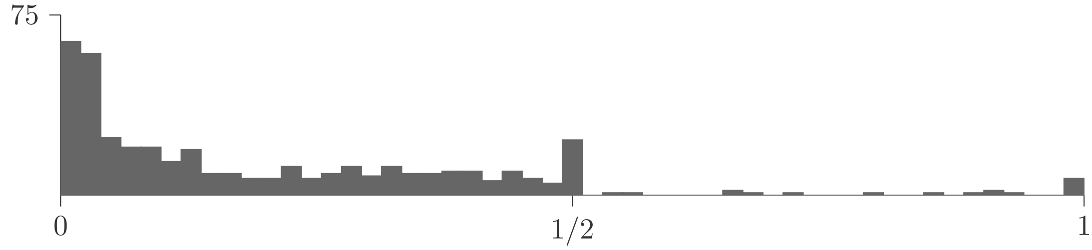
Figure 1 presents a 51-bin histogram of short ratios computed from non-zero aggregate gross short positions in debt for the eight reporting countries in CPIS between 2015 and 2023. Short positions of negligible size (with short ratios below 0.01), which constitute the majority, are excluded. The histogram shows that when non-negligible short positions are taken, short ratios rarely exceed one-half, suggesting that gross short positions are usually offset by equal or greater gross long positions within the same country, further indicating within-country heterogeneity in portfolio positions.

3.2 Aggregate Net Positions by Issuer Country

While CPIS data on aggregate gross short and long positions reveal within-country heterogeneity, they cover only a limited set of reporting countries. To broaden our analysis to 86 reporting countries, we now examine aggregate *net* holdings of foreign debt. Although net positions offer fewer insights into individual investor behavior than gross positions, they nonetheless reveal patterns that challenge the notion of widespread carry trading in debt markets. Notably, among this broader set of countries, aggregate net portfolio holdings in foreign debt are rarely negative, even in low interest-rate countries. This observation suggests that individual carry traders constitute a minority of investors in most debt markets. If a majority of investors were funding carry trades by short-selling debt in key funding currencies, we would expect to observe negative aggregate net positions with the corresponding counterparty countries, but we rarely observe this.

Table 4 presents the relative frequency and value of negative aggregate net holdings

Figure 1 – Short Ratios for Aggregate Gross Short Positions



Notes. The figure shows a 51-bin histogram of short ratios, as defined in equation (1), for eight countries reporting gross short positions in IMF CPIS data, 2015–2023. Short ratios range from 0 to 1. Zero-valued short ratios indicate long-only positions, short ratios of 0.5 indicate equal long and short positions, and short ratios above 0.5 indicate negative net positions. Short ratios below 0.01, which constitute the majority, are excluded.

of foreign debt reported in CPIS from 2001 to 2023. The data show that such positions are rare: of the 198 604 non-missing positions reported, only 129 are negative. Negative aggregate net positions constitute less than 0.1% of non-missing and less than 0.2% of non-zero positions by both count and value (rows 1 and 2). Negative aggregate net positions are rare even for Japanese debt (row 3), despite its attractiveness as a low interest-rate source of carry trade funding.

Inconsistent or under-reporting of short positions might account for the infrequency of negative aggregate net positions in CPIS data, particularly in countries that rely on custodian bank surveys (Taub, 2008). However, new standards for short position reporting implemented by the IMF in late 2009 mitigate these concerns. Table 4 actually shows a decrease in both the relative frequency and value of reported negative aggregate net positions in foreign debt after the introduction of these standards (columns 3 and 5). The fact that negative aggregate net positions remain rare suggests a fundamental feature of debt markets rather than a reporting artifact. Even if their frequency rose one hundredfold, negative aggregate net positions would still constitute fewer than one in four reported positions. We discuss further details on the reporting of short positions and the evolution of reporting standards in Appendix A.1.

3.3 Aggregate Net Positions by Issuance Currency

While CPIS provides broad coverage of country-level debt holdings, it lacks information on domestic debt holdings, on the currency composition of debt holdings by issuer, and on the holdings of non-reporting countries. To address these gaps, we estimate the currency composition of aggregate domestic and foreign debt holdings for a broad cross-section of countries, including non-reporting countries. Our estimation procedure extends existing methods for estimating domestic debt holdings and for estimating the currency composition of foreign debt holdings.

Our approach integrates two methods. Fidora et al. (2007) develop a procedure for

Table 4 – Negative Aggregate Net Holdings of International Debt

Reported Aggregate Net Positions	Of Which, Positions With Negative Values			
	in % of Total Count		in % of Total Value	
	2001–09	2010–23	2001–09	2010–23
With Non-Missing Values	0.07	0.06	0.01	0.00
With Non-Zero Values	0.17	0.12	0.01	0.00
With Non-Zero Values JPY	0.00	0.00	0.00	0.00

Notes. The table presents the relative frequency and value of negative aggregate net positions in international debt reported in IMF CPIS data between 2001 and 2023. Row 1 shows negative position counts and values as a percentage of all non-missing position counts and values. Row 2 shows negative position counts and values as a percentage of all non-zero position counts and values. Row 3 shows negative position counts and values in Japanese debt as a percentage of non-zero position counts and values in Japanese debt. Data are shown for periods before and after the 2009 IMF reporting standards change described in Appendix A.1.

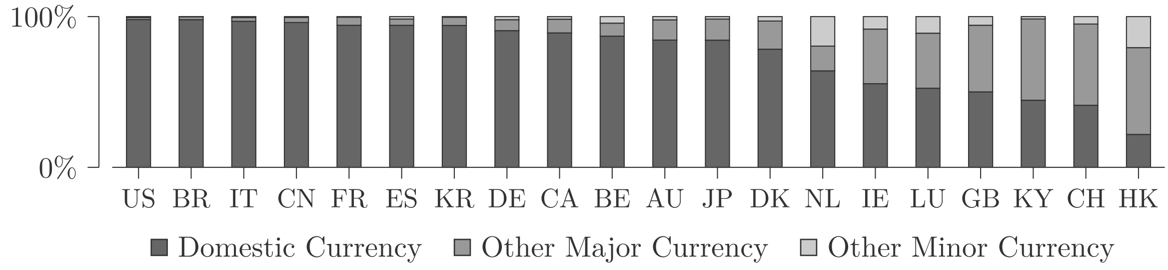
estimating domestic debt holdings as the difference between total issuance outstanding and rest-of-world holdings. Lane and Shambaugh (2010) develop a procedure for estimating the currency composition of debt holdings by issuer, using information on the currency composition of debt issuances. We extend these procedures in two ways: first, we estimate domestic debt holdings for non-reporting countries using a two-stage gravity model, and second, we refine issuance-based currency weights using a biproportional fitting procedure. These contributions are novel, as far as we know, and useful because they cover both domestic and foreign debt holdings, extend to both reporting and non-reporting countries, and produce more accurate currency composition estimates. We summarize our results below, and provide an extensive description of our methodology in Appendix A.2.[4]

Our estimates indicate a strong domestic-currency bias in aggregate net debt holdings. This finding aligns with an established literature documenting the tendency for investors to disproportionately hold domestic debt (Fidora et al., 2007; Coeurdacier and Rey, 2013) and domestic-currency debt (Burger et al., 2018; Maggiori et al., 2020). Strong domestic-currency bias in low interest-rate currencies is hard to reconcile with widespread carry trade activity. Carry trades typically involve borrowing in low interest-rate currencies and investing in high interest-rate currencies, which would result in a bias towards high interest-rate currencies, rather than the domestic-currency bias we observe in low interest-rate countries.

Figure 2 summarizes our currency composition estimates for the top twenty investor countries by total holdings. The figure shows the share of each country's debt holdings in three currency categories: domestic currency, major non-domestic currencies (USD,

[4]We do not use the capital allocation data from Maggiori et al. (2020) because it does not provide the granularity needed for our analysis. Specifically, we require data on investor country holdings by both issuer country and issuance currency to estimate portfolio returns.

Figure 2 – Currency Composition of Net Debt Holdings for Top-Twenty Investor Countries



Notes. The figure shows the currency composition for the twenty investor countries with the highest average debt holdings, 2001–2021. Holdings are classified as domestic currency, other major currency (USD, EUR, JPY, GBP, CHF), or other minor currency, referring to any other currencies. Countries are ordered by domestic currency share. Currency weights are estimated using IMF CPIS and BIS DDS and IDS databases. Appendix A.2 details the methodology.

EUR, JPY, GBP, CHF), and minor non-domestic currencies. In most countries, domestic-currency holdings dominate portfolios.[5] However, large international financial centers like the United Kingdom, the Cayman Islands, Switzerland, and Hong Kong are exceptions where major non-domestic currencies play an important role. Notably, Australia (AU) and Japan (JP) allocate similarly large shares of holdings to domestic-currency debt. This finding is hard to reconcile with widespread carry trade activity in Japan, where interest rates have been persistently low.

Our empirical analysis shows that negative aggregate net positions in debt are rare, supporting our hypothesis that carry traders are outweighed by non-carry traders. Furthermore, strong domestic currency bias suggests that most investors favor domestic over foreign currencies, particularly in low interest-rate countries. This evidence sets the stage for our theoretical model.

4 Theoretical Model

We rationalize the empirical evidence from the previous section using a two-country dynamic stochastic general equilibrium model with incomplete markets. Our model incorporates heterogeneity in risk aversion, allowing it to capture both individual carry trades and domestic bias in aggregate portfolios earning low returns. This approach helps resolve the tension we highlight between the finance literature on carry trades and the international macroeconomics literature on aggregate portfolio holdings, which

[5]We verify the high domestic currency share in the United States with a direct calculation. US debt issuance averaged 33 200 USD bn in BIS reported data during our sample period. Non-US investors held 4 863 USD bn (14.6% of total issuance) and global central banks held 2 087 USD bn (6.3% of total issuance), based on IMF data. This implies 26 250 USD bn in US holdings of domestic debt (assumed USD-denominated). US investors held 2 451 USD bn in foreign-issued debt, with an IMF-reported 78% denominated in USD. Total US holdings in USD were therefore 28 160 USD bn (26 250 bn domestic + 1 910 bn foreign-issued in USD), representing 98.1% of total US debt holdings (28 700 USD bn).

representative-agent models cannot resolve.

In our model, agents tailor their consumption to their individual risk preferences: agents with higher risk tolerance choose higher expected consumption with higher variance, while agents with lower risk tolerance choose lower expected consumption with lower variance. Agents achieve these consumption patterns by endogenously choosing speculative, hand-to-mouth, or hedged portfolio positions that raise or lower the variance of their incomes.

The model features a determinate non-stochastic steady state and stationary dynamics for both individual agents and the aggregate economy. The common tools used to address issues of indeterminacy and non-stationarity in representative-agent models in international macroeconomics are not well suited to addressing these issues at both the individual and aggregate levels in our heterogeneous-agent model. We therefore turn to a less common tool: incorporating wealth into the utility function.

In the following paragraphs, we introduce the primitive assumptions of our model and derive a series of propositions that demonstrate how heterogeneity in risk aversion can simultaneously produce individual carry trade positions and domestic bias in aggregate portfolios earning low returns. The model allows for exact aggregation across agents in the non-stochastic steady state and approximate aggregation locally around the steady state. We use perturbation methods to derive approximate closed-form solutions for the real exchange rate, real returns, and both individual and aggregate consumption, real wealth, and portfolio holdings, underscoring the model's tractability.

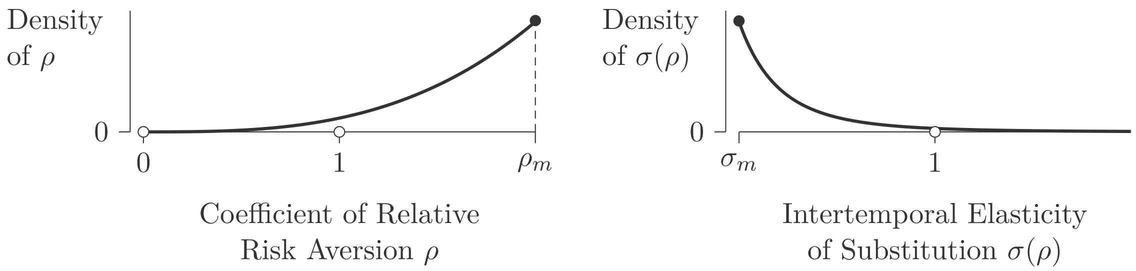
4.1 Model Primitives

There are two countries, home H and foreign F. To avoid repeating home and foreign versions of equations throughout the text, we use $i, j, k \in \{H, F\}$ to index countries and currencies and write all equations in terms of these indices. Equations with multiple indices should be understood to hold for any combination of index values unless we explicitly indicate otherwise.

A continuum of agents exists in each country. We denote the set of agents $\mathcal{H}_i$ in country i and we assume a measure of one for the set of agents. Each agent is uniquely identified within country by its coefficient of relative risk aversion, denoted ρ, so we use ρ as the index for agents. Each country has an identical distribution of risk aversion across agents, so each agent has a counterpart abroad with an identical coefficient of relative risk aversion.

We define an inverse Pareto density function to characterize the distribution of risk

Figure 3 – Distributions of Risk Aversion and Intertemporal Substitution



Notes. The figures plot the distribution of the coefficient of risk aversion ρ across individual agents (left) and the corresponding distribution of the elasticity of intertemporal substitution $\sigma(\rho)$ across individual agents (right) for an illustrative calibration of $\kappa = 4$, $\rho_m = 2$, and $\sigma_m = 1/2$. The point $\rho = \sigma(\rho) = 1$ is excluded from the support of both distributions. The two density functions plotted here represent equivalent heterogeneity across agents.

aversion across agents.[6] The density function, denoted $f(\rho)$ and given by

$$f(\rho) = \frac{\kappa}{\rho}\left(\frac{\rho}{\rho_m}\right)^{\kappa}, \tag{2}$$

has positive support over $\mathcal{R} = (0, \rho_m]$, where $\kappa > 1$ and $\rho_m > (\kappa^2 + \kappa)/(\kappa^2 - 1)$ are shape and scale parameters, respectively.[7] These parameter restrictions ensure that aggregate risk aversion is well-defined and sufficiently high. We illustrate the density function in the left panel of Figure 3. The density rises from zero as ρ approaches ρ_m from below, forming a left tail of agents with decreasing coefficients of risk aversion. Agents near the tip of the tail are nearly risk neutral; these will be the carry traders in our model.

Agent ρ in country i maximizes the expected present value of lifetime utility over consumption and wealth. Let $U_{it}(\rho)$ denote the agent's expected present value of lifetime utility,

$$U_{it}(\rho) = \mathrm{E}_t\left[\sum_{s=t}^{\infty} \beta^{s-t}\Big(U_{Cis}(\rho) + U_{Wis}(\rho)\Big)\right], \tag{3}$$

where $\beta \in (0.5, 1)$ denotes the subjective discount factor, and where $U_{Cit}(\rho)$ and $U_{Wit}(\rho)$ denote the agent's period utility from consumption and real wealth, respectively. We assume constant relative risk aversion for $U_{Cit}(\rho)$ and constant absolute risk aversion for

[6] Our results do not depend on the specific distribution that we assume. For example, uniform, triangular, normal, or log-normal distributions also work. We only require the distribution to support a small measure of sufficiently risk-tolerant agents and to imply a sufficient level of aggregate risk aversion. The inverse Pareto distribution does, however, have some empirical support: Kimball et al. (2008) estimate a qualitatively similar empirical distribution from a number of large-scale surveys conducted in the United States and internationally, where respondents state their preferences over risky gambles related to employment and lifetime earnings. On the other hand, Calvet et al. (2021) estimate a more symmetric empirical distribution of risk aversion using Swedish administrative data and assuming Epstein-Zin preferences.

[7] The agent with exactly $\rho = 1$ is measure zero, and assumed to have log utility.

$U_{Wit}(\rho)$,

$$U_{Cit}(\rho) = \frac{1}{1-\rho}\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{1-\rho} \quad \text{and} \quad U_{Wit}(\rho) = -\frac{\theta}{\rho}\left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho}, \tag{4}$$

where $C_{it}(\rho)$ denotes the agent's consumption basket, with subscript i denoting the agent's country of residence, and $W^i_{it}(\rho)$ denotes the agent's real wealth, with subscript i denoting the agent's country of residence and superscript i denoting the numéraire basket. Agents have internal habits set at the non-stochastic steady-state levels of consumption and real wealth, where the absence of time subscripts on these variables indicates their steady state levels.[8]

The parameter θ governs the importance of real wealth as a direct source of utility. We introduce wealth into the utility function for practical reasons: it ensures stationarity in the model even for arbitrarily small values of θ that have no significant impact on equilibrium outcomes. Throughout the paper, we assume $\theta \in [0, \epsilon)$, where ϵ is a small positive number. Nevertheless, wealth-in-utility can be economically justified by factors like social status and is used as a modeling device in recent studies by Michaillat and Saez (2021), He et al. (2023), Michau et al. (2023), and Zhao (2023). Unlike these papers, we show that wealth-in-utility achieves stationarity while preserving aggregability in the presence of heterogeneity.

The consumption utility function in (4) directly links the coefficient of relative risk aversion to the elasticity of intertemporal substitution for individual agents, meaning that heterogeneity in the former implies heterogeneity in the latter. Letting $\sigma(\rho)$ denote the elasticity intertemporal of substitution with respect to consumption for agent ρ, the utility function in (4) implies

$$\sigma(\rho) = 1/\rho\,. \tag{5}$$

The inverse Pareto density function that we assume for ρ in (2) implies a Pareto density function for $\sigma(\rho)$ with shape parameter κ and scale parameter $\sigma_m = 1/\rho_m$. The Pareto density function falls in $\sigma(\rho)$, forming a right tail of agents with increasing elasticities of intertemporal substitution. We illustrate the density function for $\sigma(\rho)$ in the right panel of Figure 3.

Each agent's budget constraint ensures that real expenditure on consumption and bond holdings each period does not exceed the agent's real endowment income plus the gross real return on bond holdings from the previous period. We write the constraint as

[8] We choose these functional forms for tractability. Internal habits simplify the non-stochastic steady state and the aggregation of approximate equilibrium conditions. CARA utility over real wealth simplifies our specification of bond markets, because it admits the zero and negative values of real wealth that arise when bonds are in zero net supply. The model remains tractable if we instead specify CRRA utility over real wealth and bonds in positive net supply.

an equality,

$$C_{it}(\rho) + B^i_{iit}(\rho) + B^i_{ijt}(\rho) = \frac{P^i_{Cit}}{P^i_{it}} Y_{it}(\rho) + R^i_{it} B^i_{iit-1}(\rho) + R^i_{jt} B^i_{ijt-1}(\rho)\,, \quad i \neq j\,, \tag{6}$$

where $B^i_{ijt}(\rho)$ denotes the real value of bond holdings for agent ρ, with subscripts i and j denoting the agent's country of residence and the bond's country of issuance, respectively, and superscript i denoting the numéraire basket; P^i_{Cit} and P^i_{it} denote consumption good and basket prices, respectively, with subscript i denoting the country and superscript i denoting the numéraire currency; $Y_{it}(\rho)$ denotes agent ρ's real endowment, with subscript i denoting the agent's country of residence; and R^i_{jt} denotes the gross real return, with subscript j denoting the bond's country of issuance and superscript i denoting the numéraire basket. We state the equivalent nominal budget constraint in Appendix B.2.

We define agents' real wealth as the real value of their domestic and international nominal bond holdings, expressed in a common numéraire good. That is,

$$W^i_{it}(\rho) = B^i_{iit}(\rho) + B^i_{ijt}(\rho)\,, \quad i \neq j\,. \tag{7}$$

Nominal bonds are single-period assets that pay one unit of the issuer's currency at maturity with certainty. We define agents' real bond holdings and the gross real return as

$$B^i_{ijt}(\rho) = \frac{A_{ijt}(\rho) P^i_{Bjt}}{P^i_{it}} \quad \text{and} \quad R^i_{jt} = \frac{S_{ijt}/P^i_{Bjt-1}}{P^i_{it}/P^i_{it-1}}\,, \tag{8}$$

respectively, where $A_{ijt}(\rho)$ denotes the quantity of bonds held by agent ρ, with subscripts i and j denoting the bond holder's country and the bond's country of issuance, respectively; P^i_{Bjt} denotes the nominal bond price, with subscript j denoting the bond's country of issuance and superscript i denoting the numéraire currency; and S_{ijt} denotes the nominal exchange rate in units of currency i per unit of currency j, with $S_{iit} = 1$ by definition.

Each period, all agents within a country receive an identical endowment, which is a positive random variable. Since the measure of agents in each country equals one, each agent's endowment equals the aggregate endowment,

$$Y_{it}(\rho) = Y_{it}\,. \tag{9}$$

We normalize the steady-state endowment to one, $Y_i = 1$, where the absence of a time subscript indicates the steady state.

Endowment goods trade internationally, and agents choose the quantity of each country's endowment good to consume. Agents differentiate between domestic and international endowment goods, and prefer variety in their consumption. We define each agent's consumption basket as

$$C_{it}(\rho) = \gamma C_{iit}(\rho)^{\alpha_{ii}} C_{ijt}(\rho)^{\alpha_{ij}}\,, \quad i \neq j\,, \tag{10}$$

where $C_{ijt}(\rho)$ denotes the quantity of good j that agent ρ in country i consumes, and where the parameter α_{ij} denotes country i's expenditure share on the endowment good of country j. Expenditure shares sum to one, $\alpha_{ii}+\alpha_{ij} = 1$, and we assume that the shares exogenously embed domestic bias in agents' consumption preferences, $\alpha_{ii} \in (1/2, 1)$. Agents decide the composition of their consumption baskets after first deciding on their total consumption expenditure in a two-stage budgeting procedure. Their total consumption expenditure therefore constrains their expenditure on domestic and international goods,

$$P^i_{it} C_{it}(\rho) = P^i_{Cit} C_{iit}(\rho) + P^i_{Cjt} C_{ijt}(\rho)\,, \quad i \neq j\,, \tag{11}$$

where P^j_{Cit} denotes the price of good i in numéraire currency j.

Because goods and nominal bonds are differentiated across countries, each trades in a separate market with a separate clearing condition. Market clearing requires that aggregate demand across countries equals aggregate supply,

$$C_{iit} + C_{jit} = Y_{it} \quad \text{and} \quad B^i_{iit} + B^i_{jit} = 0\,, \quad i \neq j\,, \tag{12}$$

where we assume zero net supply of each country's nominal bonds.

The law of one price holds for goods and nominal bonds, so goods prices across countries and bond prices across countries must be equalized when expressed in a common numéraire currency. That is,

$$P^i_{Cit} = S_{ijt} P^j_{Cit} \quad \text{and} \quad P^i_{Bit} = S_{ijt} P^j_{Bit}\,. \tag{13}$$

Because consumption baskets differ across countries, consumption basket prices also differ across countries and we define the real exchange rate as the ratio of consumption basket prices expressed in a common numéraire currency,

$$Q_{ijt} = S_{ijt} P^j_{jt} \Big/ P^i_{it}\,, \tag{14}$$

where Q_{ijt} denotes the real exchange rate in units of consumption basket i per unit of consumption basket j, with $Q_{iit} = 1$ by definition.

A quantity equation determines the price level in each country, and we assume that each country's money supply is a positive random variable. We assume symmetric money supplies across countries in the non-stochastic steady state, when expressed in a common numéraire currency,

$$M^i_{it} = Y_{it} P^i_{it} \quad \text{and} \quad M^i_i = S_{ij} M^j_j \,, \tag{15}$$

where M^j_{it} is the supply of money in country i in numéraire currency j, and where the absence of the time subscript indicates the steady state.

The model features four sources of uncertainty—home and foreign goods endowments and home and foreign money supplies—and only two nominal bonds, making financial markets incomplete. The logarithms of endowments and money supplies have positive and finite means, and each follows an independent and identically distributed process over time. We assume a variance-covariance matrix with positive and finite elements, where domestic covariances exceed cross-country covariances. This structure is detailed in Appendix B.1 and helps to ensure that portfolios with domestic bias earn lower expected real returns than portfolios with international bias.[9]

4.2 Model Propositions

We solve the model in two stages: first for the aggregate economy and then for individual agents, distinguishing between non-portfolio and portfolio problems at each stage. The model is tractable precisely because it separates into agent and aggregate, as well as non-portfolio and portfolio problems.

The propositions that follow provide a roadmap to our theoretical model. In Section 4.2.1 we state first-order optimality conditions. In Section 4.2.2, Propositions 1 and 2 characterize the non-stochastic steady state, where heterogeneity vanishes, contributing to the model's tractability. In Section 4.2.3, Propositions 3 and 4 introduce the concept of the aggregation wedge and demonstrate how first-order Taylor expansions of agent-specific Euler equations can be aggregated around the non-stochastic steady state. In Section 4.2.4, Proposition 5 establishes stationarity for the aggregate economy, and we solve for aggregate consumption, real wealth, and the real exchange rate. In Section 4.2.5, Propositions 6 through 9 introduce the portfolio valuation multiplier and establish sufficient conditions for domestic bias in aggregate portfolio holdings and a negative cross-country difference in real returns. In Section 4.2.6, we solve for agent-specific consumption and real wealth. In Section 4.2.7, Proposition 10, the culminating result of our model, states sufficient conditions for the existence of carry traders in economies with domestic bias in aggregate portfolios earning low returns.

[9]Our core results extend to more general variance-covariance structures, described in Appendix B.1, that can produce persistent differences in interest rates across countries. We adopt the block-symmetric structure to simplify exposition in order to focus on within-country heterogeneity in portfolio holdings. See Hassan and Zhang (2021) for a review of cross-country interest rate differentials.

4.2.1 Utility Maximization.

Agents solve a two-stage utility maximization problem. In the first stage, agents maximize expected discounted lifetime utility in (3) by choosing quantities of the consumption basket to consume, and by constructing a portfolio of home and foreign nominal bonds. For agent ρ in country i, the first stage of the utility maximization problem yields the Euler equations,

$$\beta\,\mathrm{E}_t\left[\left(\frac{C_{it+1}(\rho)}{C_i(\rho)}\right)^{-\rho}\frac{1}{C_i(\rho)}R^i_{jt+1}\right]=\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho}\frac{1}{C_i(\rho)}-\theta\left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho}. \tag{16}$$

If the wealth-in-utility parameter θ is positive, real wealth directly raises utility, in addition to the traditional indirect effect through consumption. A rise in present consumption that lowers real wealth will simultaneously lower utility from real wealth, and agents forming optimal consumption plans will account for this effect by subtracting the marginal utility of real wealth from the marginal utility of consumption in their Euler equations. This trade-off determines an optimal level of real wealth and ensures stationarity.

In the second stage, agents choose quantities of home and foreign goods to include in their consumption baskets, subject to the plans they formed in the first stage of the problem. The second-stage problem yields demand equations for the home and foreign goods,

$$C_{ijt}(\rho)=\alpha_{ij}\frac{P^i_{it}}{P^i_{Cjt}}C_{it}(\rho)\,. \tag{17}$$

The demand functions in (17) can be rearranged to equate α_{ij} with the expenditure share on the good from country j for agents in country i.

Finally, the expenditure constraints in (11) and goods demands in (17) from agents' second-stage utility maximization problems together with the definition of the consumption basket in (10) imply an optimal price index. For the consumption basket in country i, the price index in numéraire currency k is

$$P^k_{it}=\left(P^k_{Cit}\right)^{\alpha_{ii}}\left(P^k_{Cjt}\right)^{\alpha_{ij}}\,,\quad i\neq j\,. \tag{18}$$

We derive the Euler equation in (16), the goods demands in (17), and the price index in (18) in Appendix B.3.

4.2.2 Non-Stochastic Steady State.

We now derive the non-stochastic steady-state equilibrium of the model, where all variables are constant and equal to their expected values. In the steady state, the model simplifies considerably because all heterogeneity vanishes.

Evaluating the agent-specific Euler equations in (16) at the non-stochastic steady state, we obtain

$$\theta C_i(\rho) = 1 - \beta R_j^i \,, \tag{19}$$

where the absence of time subscripts indicates the steady state. The left side of (19) is independent of j, so the steady-state real return on the home bond equals the steady-state real return on the foreign bond when returns are computed using a common numéraire currency, $R_i^i = R_j^i$.

If we set $\theta = 0$, real wealth drops out of the utility function. The agent-specific Euler equations in (19) simplify to $R_i^i = R_j^i = 1/\beta$ and impose no optimality conditions on steady-state consumption for individual agents. Without such conditions, consumption is only constrained by agents' budgets, but budget constraints alone cannot determine both consumption and real wealth, so the steady state is indeterminate. Assuming an exogenous steady-state distribution of real wealth across agents would not fully repair the model because the model is also non-stationary when we set $\theta = 0$, as we later show. For these reasons, we focus on positive values of θ in the neighborhood of zero, $\theta \in [0, \epsilon)$.

Numerous alternatives to wealth-in-utility have been proposed to induce determinacy and stationarity in representative-agent models. Common alternatives include endogenous subjective discount factors, portfolio holding or adjustment costs, and overlapping generations, as discussed by Schmitt-Grohé and Uribe (2003) and Ghironi (2006). We find wealth in the utility function to be the simplest and most tractable device in our setting with heterogeneity in risk aversion, because other devices either hinder aggregation or introduce an incidental second dimension of heterogeneity to the model.

Because agents differ only in their aversion to risk and because risk is absent from the non-stochastic steady state, the steady-state distribution of consumption and real wealth across agents is degenerate, in the sense that all agents consume equal quantities and hold zero real wealth. This fact leads to a particularly simple steady-state solution for agent and aggregate non-portfolio variables and allows us to state the following proposition, which we prove in Appendix B.4.1.

Proposition 1 (Non-Stochastic Steady-State Non-Portfolio Equilibrium)**.** *Heterogeneity in risk aversion vanishes from the non-portfolio equilibrium conditions of the model in the non-stochastic steady state. The distribution of non-portfolio variables across agents is endogenously degenerate.*

Agent-specific and aggregate steady-state consumption baskets and goods demands are given by

$$C_i(\rho) = C_i = 1 \quad \textit{and} \quad C_{ij}(\rho) = C_{ij} = \alpha_{ij} \,, \tag{20}$$

steady-state price indices for consumption baskets, prices for consumption goods, and the *real exchange rate are given by*

$$P_i^j = P_{Ci}^j = M_i^j \quad \textit{and} \quad Q_{ij} = 1\,, \tag{21}$$

and agent-specific and aggregate steady-state real wealth and real returns on nominal bonds are given by

$$W_i^j(\rho) = W_i^j = 0 \quad \textit{and} \quad R_i^j = (1-\theta)/\beta\,. \tag{22}$$

Because real returns on domestic and international nominal bonds are equal in the steady state, we drop country and currency superscripts and henceforth write R.

We now turn to steady-state portfolios. In contrast to the non-portfolio steady state, the portfolio steady state does depend on agent and aggregate risk aversion. We derive steady-state portfolio solutions by extending to our heterogeneous-agent setting a method developed by Samuelson (1970) and Devereux and Sutherland (2011) for obtaining lower-order approximations of portfolio holdings from higher-order approximations of equilibrium conditions. Specifically, we derive order-zero approximate individual and aggregate portfolio holdings from second-order approximate equilibrium conditions. The latter conditions involve variance and covariance terms, which each agent evaluates differently according to their individual risk aversion. This fact produces a non-degenerate distribution of portfolio holdings across agents, which we later characterize analytically. For now, we provide a limited characterization in the following proposition, which we prove in Appendix B.4.2.

Proposition 2 (Non-Stochastic Steady-State Portfolio Equilibrium)**.** *Agent-specific and aggregate steady-state portfolio holdings satisfy*

$$B_{ii}^i(\rho) + B_{ij}^i(\rho) = B_{ii}^i + B_{ij}^i = 0\,, \quad i \neq j\,. \tag{23}$$

Additionally, aggregate steady-state portfolio holdings satisfy

$$B_{ii}^i = B_{jj}^i\,, \tag{24}$$

while no such condition holds for agents, because bond markets must only clear in aggregate.

We later show that heterogeneity in agents' steady-state portfolios contributes to heterogeneity in consumption and real wealth dynamics through portfolio valuation effects. At the same time, the absence of heterogeneity in the non-portfolio steady state greatly simplifies aggregation, both in and around the steady state. We exploit this aggregability extensively in the following sections.

4.2.3 Model Aggregability

We aggregate variables across agents by integrating the product of agent variables and the density function $f(\rho)$,

$$\begin{gathered} Y_{it} = \int_{\mathcal{R}} Y_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad C_{it} = \int_{\mathcal{R}} C_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad C_{ijt} = \int_{\mathcal{R}} C_{ijt}(\rho) f(\rho)\,\mathrm{d}\rho\,, \\ W_{it}^{j} = \int_{\mathcal{R}} W_{it}^{j}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad \text{and} \quad B_{ijt}^{k} = \int_{\mathcal{R}} B_{ijt}^{k}(\rho) f(\rho)\,\mathrm{d}\rho\,, \end{gathered} \tag{25}$$

where the absence of an agent index indicates an aggregate variable. The set of agents in each country has a measure of one, so our model makes no distinction between aggregates and averages.

We apply the same procedure to agents' coefficients of risk aversion and intertemporal elasticities of substitution to obtain aggregates of these parameters. The simple inverse relationship between the agent-specific parameters in (5) breaks down in aggregate, due to a wedge that heterogeneity produces, as we emphasize in the following proposition and prove in Appendix B.6.2.

Proposition 3 (Aggregate Preference Parameters)**.** *We define the aggregate coefficient of risk aversion $\bar{\rho}$ and elasticity of intertemporal substitution $\bar{\sigma}$ as*

$$\bar{\rho} = \int_{\mathcal{R}} \rho f(\rho)\,\mathrm{d}\rho \quad \textit{and} \quad \bar{\sigma} = \int_{\mathcal{R}} \sigma(\rho) f(\rho)\,\mathrm{d}\rho\,, \tag{26}$$

respectively. Given the density function in (2)*, it follows that*

$$\bar{\sigma} = \omega/\bar{\rho}\,, \quad \textit{with} \quad \omega = \kappa^2/(\kappa^2 - 1)\,, \tag{27}$$

where ω denotes the aggregation wedge. The wedge exceeds one for finite values of the shape parameter κ, and approaches one in the limit as κ approaches infinity. As κ rises, the cross-sectional distribution of risk aversion collapses around the scale parameter ρ_m.

The aggregation wedge ω decouples aggregate risk preferences from aggregate intertemporal substitutability, offering an intermediate degree of flexibility between time-separable preferences with constant relative risk aversion and the recursive preferences of Epstein and Zin (1989).

Aggregating variables and parameters over agents is straightforward, but aggregating equilibrium conditions over agents is not. Complications arise in equilibrium conditions when an agent variable cannot be additively separated from the agent-specific coefficient of relative risk aversion. In particular, the agent Euler equations in (16) suffer from these complications and are not easily aggregable. However, in the non-stochastic steady state, heterogeneity vanishes and aggregating equilibrium conditions over agents becomes trivial. The absence of heterogeneity in the steady state makes perturbation methods particularly useful in our setting.

To approximate the model, we use Taylor expansions around the non-stochastic steady state. For positive variables like consumption baskets, goods demands, endowments, money supplies, prices, and exchange rates, we use hats to denote logarithmic deviations from the steady state. Since portfolio holdings and real wealth are not always positive, we expand these variables in levels around their steady-state values. For real wealth, hats indicate deviations relative to the steady-state endowment, $\hat{W}^j_{it}(\rho) = \big(W^j_{it}(\rho) - W^j_i(\rho)\big)/Y_j$, because steady-state real wealth is zero. For real returns, we also expand in levels, using hats to denote deviations relative to the steady-state level, $\hat{R}^j_{it} = \big(R^j_{it} - R\big)/R$.

The agent Euler equations in (16) present the only challenge to aggregation in our model, as they are the only equilibrium conditions where the agent-specific coefficient of relative risk aversion appears explicitly. However, with internal habits, this coefficient is additively separable from agent variables in first-order approximations, making the Euler equations first-order aggregable. We highlight this convenient properly in the following proposition, which we prove in Appendix B.6.3.

Proposition 4 (First-Order Aggregate Euler Equations)**.** *First-order Taylor expansions of agent Euler equations in* (16) *around the non-stochastic steady state are given by*

$$\hat{C}_{it}(\rho) = \theta \hat{W}^i_{it}(\rho) + \beta R\, \mathrm{E}_t\left[\hat{C}_{it+1}(\rho) - \frac{1}{\rho}\hat{R}^i_{jt+1}\right] + O(\epsilon^2)\,, \tag{28}$$

where the inverse of the agent-specific coefficient of relative risk aversion multiples the real return, an aggregate variable. Multiplying the agent-specific Euler equations by the density function in (2) *and integrating yields the aggregate Euler equations*

$$\hat{C}_{it} = \theta \hat{W}^i_{it} + \beta R\, \mathrm{E}_t\left[\hat{C}_{it+1} - \frac{\omega}{\bar{\rho}}\hat{R}^i_{jt+1}\right] + O(\epsilon^2)\,, \tag{29}$$

where the aggregation wedge ω defined in (27) *summarizes the first-order effect of heterogeneity on the aggregate economy.*

The agent-specific coefficient of relative risk aversion does not appear directly in the remaining agent-specific equilibrium conditions, and first-order Taylor expansions of these conditions allow for straightforward first-order aggregation. The first-order Taylor expansion of the agent-specific budget constraints in (6) and the corresponding aggregate budget constraints are given by

$$\begin{aligned}\hat{W}^i_{it}(\rho) + \hat{C}_{it}(\rho) &= R\hat{W}^i_{it-1}(\rho) + RB^i_{ii}(\rho)\Big(\hat{R}^i_{it} - \hat{R}^i_{jt}\Big) \\ &\quad + \hat{P}^i_{Cit} - \hat{P}^i_{it} + \hat{Y}_{it}(\rho) + O(\epsilon^2) \quad \text{and} \\ \hat{W}^i_{it} + \hat{C}_{it} &= R\hat{W}^i_{it-1} + RB^i_{ii}\Big(\hat{R}^i_{it} - \hat{R}^i_{jt}\Big) \\ &\quad + \hat{P}^i_{Cit} - \hat{P}^i_{it} + \hat{Y}_{it} + O(\epsilon^2)\,, \quad i \neq j\,.\end{aligned} \tag{30}$$

The first-order Taylor expansion of agent-specific goods demands in (17) and the corresponding aggregate goods demands are given by

$$\hat{C}_{ijt}(\rho) = \hat{P}^j_{it} - \hat{P}^j_{Cit} + \hat{C}_{it}(\rho) + O(\epsilon^2) \quad \text{and} \quad \hat{C}_{ijt} = \hat{P}^j_{it} - \hat{P}^j_{Cit} + \hat{C}_{it} + O(\epsilon^2)\,. \tag{31}$$

The remaining equilibrium conditions of the model depend solely on aggregate variables and require no aggregation. Their first-order Taylor expansions are standard and we provide them in Appendix B.5.

4.2.4 The Aggregate Non-Portfolio Problem

We find it convenient to solve the model first for cross-country differences. Using these solutions and market clearing conditions, we then derive country-specific solutions for aggregate variables.

The aggregate equilibrium conditions derived from first-order aggregation of agent-specific equilibrium conditions together with first-order approximations of the remaining of equilibrium conditions of the model produce a system of aggregate equations with no agent-specific variables. This aggregate system can be solved independently from the agent-specific equilibrium conditions of the model.

We solve the aggregate system first in terms of cross-country differenced aggregate consumption and real wealth. These solutions then determine the real exchange rate. To see that cross-country differences suffice, combine goods market clearing conditions in (12) with aggregate goods demand equations in (31) to obtain

$$\phi\hat{Y}_{i-jt} = \Big(1 - \phi^2\Big)\hat{Q}_{ijt} + \phi^2\hat{C}_{i-jt} + O(\epsilon^2) \quad \text{with} \quad \phi \equiv \alpha_{ii} - \alpha_{ij}\,, \tag{32}$$

where the subscript $i - j$ denotes a cross-country difference. The parameter ϕ measures the strength of preferences for the domestic good, is symmetric across countries, and lies between zero and one, $\phi \in (0, 1)$. Equation (32) states that the real exchange rate and cross-country difference in consumption adjust to clear international goods markets. The degree to which adjustment occurs through the real exchange rate or consumption depends on the strength of preferences for domestic goods and the tolerance of agents for consumption fluctuations.

The market clearing condition in (32) is one of three equations forming the cross-country differenced aggregate system. The other two are the cross-country differenced aggregate Euler equation and the cross-country differenced aggregate budget constraint. These equations depend on deflated nominal variables—specifically, aggregate real wealth and the real return on nominal bonds——and it is useful to express these in a common numéraire currency. Using definitions from (7) and (8), and noting that steady-state aggregate real wealth equals zero (Proposition 2), we obtain

$$\begin{aligned} \hat{W}^i_{i-jt} &= \hat{W}^i_{it} - \hat{W}^i_{jt} = \hat{W}^i_{it} - \hat{W}^j_{jt} + O(\epsilon^2) \quad \text{and} \\ \hat{R}^i_{i-jt} &= \hat{R}^i_{it} - \hat{R}^i_{jt} = \hat{R}^i_{it} - \hat{R}^j_{jt} - \left(\hat{Q}_{ijt} - \hat{Q}_{ijt-1}\right) + O(\epsilon^2)\,, \end{aligned} \tag{33}$$

where $\hat{W}^k_{i-jt}$ denotes the cross-country difference in aggregate real wealth and $\hat{R}^k_{i-jt}$ denotes the cross-country difference in real returns on nominal bonds, both in numéraire basket k.

With this notation in place, we write the cross-country differenced aggregate budget constraint and Euler equation as

$$\begin{aligned} \hat{W}^i_{i-jt} + \hat{C}_{i-jt} &= R\hat{W}^i_{i-jt-1} - \frac{1-\phi}{\phi}\hat{Q}_{ijt} + \hat{Y}_{i-jt} + R\hat{V}^i_{i-jt} + O(\epsilon^2) \quad \text{and} \\ \hat{C}_{i-jt} - \theta\hat{W}^i_{i-jt} &= \beta R\,\mathrm{E}_t\left[\hat{C}_{i-jt+1} - \frac{\omega}{\bar{\rho}}\left(\hat{R}^i_{i-jt+1} + \hat{Q}_{ijt+1} - \hat{Q}_{ijt}\right)\right] + O(\epsilon^2)\,, \end{aligned} \tag{34}$$

respectively, where we introduce the cross-country differenced aggregate portfolio valuation effect $\hat{V}^i_{i-jt}$, defined as

$$\hat{V}^i_{i-jt} \equiv B^i_{ii-ji}\hat{R}^i_{i-jt} + O(\epsilon^2) \quad \text{with} \quad B^i_{ii-ji} \equiv B^i_{ii} - B^i_{ji}\,. \tag{35}$$

The portfolio valuation effect is endogenous, but behaves as an independent and identically distributed random variable, as shown by Devereux and Sutherland (2011). Consequently, we can temporarily treat the effect as if it were exogenous and derive intermediate solutions in terms of it. Later, we can use these intermediate solutions together with second-order Taylor expansions of the Euler equations in (16) to derive final solutions.

Using the market clearing condition in (32) to eliminate the real exchange from the cross-country differenced aggregate Euler equation and budget constraint in (34), we obtain a system of two first-order approximate difference equations,

$$\mathrm{E}_t\left[\hat{\boldsymbol{Z}}_{i-jt+1}\right] = \boldsymbol{\mathcal{E}}^{(-)}_{ZZ}\hat{\boldsymbol{Z}}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{ZY}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{ZV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{36}$$

where $\hat{\boldsymbol{Z}}_{i-jt} = \begin{bmatrix}\hat{W}^i_{i-jt-1} & \hat{C}_{i-jt}\end{bmatrix}'$ denotes the (2×1) vector of cross-country differences in aggregate real wealth and aggregate consumption and where $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$, $\boldsymbol{\mathcal{E}}^{(-)}_{ZV}$, and $\boldsymbol{\mathcal{E}}^{(-)}_{ZY}$ denote matrices of partial elasticities that depend only on the parameters of the model. The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$, for example, is (2×2) with elements $\mathcal{E}^{(-)}_{mn}$, where $m, n \in \{W, C\}$. We provide a derivation of the cross-country differenced aggregate system in (36) in Appendix B.7, along with expressions for the full set of partial elasticities.

The cross-country differenced aggregate system in (36) has one non-predetermined variable and admits a unique non-explosive solution if the matrix of partial elasticities $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ has exactly one eigenvalue that lies strictly outside the unit circle, satisfying the conditions of Blanchard and Kahn (1980). If the second eigenvalue lies strictly inside the unit circle, the system is stationary.

Setting $\theta = 0$ removes wealth from the utility function and the aggregate system joins the class of non-stationary incomplete markets models discussed by Schmitt-Grohé and Uribe (2003) and Ghironi (2006). This knife-edge non-stationarity serves as a benchmark: by considering the neighborhood around $\theta = 0$, we establish a condition for stationarity, stated in the following proposition and proved in Appendix B.7.2.

Proposition 5 (Stationarity of the Cross-Country Differenced Aggregate System)**.** *Let $\theta \in [0, \epsilon)$, where ϵ is a small positive number. Then the matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ has two distinct real-valued eigenvalues, $\lambda^{(-)}_{ZW}$ and $\lambda^{(-)}_{ZC}$. Furthermore, $\lambda^{(-)}_{ZW}\big|_{\theta=0} = 1/\beta$ and $\lambda^{(-)}_{ZC}\big|_{\theta=0} = 1$, and*

$$\partial\lambda^{(-)}_{ZW}/\partial\theta\,\Big|_{\theta=0} > 0 \quad \text{and} \quad \partial\lambda^{(-)}_{ZC}/\partial\theta\,\Big|_{\theta=0} < 0\,. \tag{37}$$

Hence, there exists a $\theta > 0$ that yields a unique and stationary rational expectations solution to the cross-country differenced aggregate system in (36)*.*

The cross-country differenced aggregate system admits a unique and stationary solution when the wealth-in-utility parameter θ takes a small positive value, and we proceed under this assumption.[10]

To derive intermediate solutions to the aggregate system in (36), we first factorize the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ into a diagonal matrix of eigenvalues and a matrix of eigenvectors, decoupling the two difference equations in the system. We then impose the condition that the endogenous variables in the decoupled equations do not "explode too fast," as in Blanchard and Kahn (1980). This procedure yields intermediate solutions in terms of lagged aggregate real wealth, aggregate real portfolio valuation effects, and aggregate endowments in cross-country differences. Coefficients in the intermediate solution represent semi-partial elasticity—semi-partial because they fail to capture the effects of exogenous variables that act on aggregate real wealth and aggregate consumption indirectly through portfolio valuation effects.

Our intermediate solution to the cross-country differenced aggregate system in (36) is

[10] We remark that the aggregate cross-country differenced system also admits a unique and stationary solution when θ is negative and real wealth has a direct negative effect on utility.

given by

$$\begin{bmatrix} \hat{W}^i_{i-jt} \\ \hat{C}_{i-jt} \end{bmatrix} = \boldsymbol{\eta}^{(-)}_{ZW} \hat{W}^i_{i-jt-1} + \boldsymbol{\eta}^{(-)}_{ZY} \hat{Y}_{i-jt} + \boldsymbol{\eta}^{(-)}_{ZV} \hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{38}$$

where $\boldsymbol{\eta}^{(-)}_{ZW}$, $\boldsymbol{\eta}^{(-)}_{ZY}$, and $\boldsymbol{\eta}^{(-)}_{ZV}$ denote matrices of semi-partial elasticities that depend only on the parameters of the model.[11]

To obtain intermediate solutions for country-specific aggregate real wealth and consumption, we use the intermediate solutions for cross-country differences in (38) together with first-order Taylor expansions of the market clearing conditions in (12),

$$\hat{C}_{i+jt} = \hat{Y}_{i+jt} + O(\epsilon^2) \quad \text{and} \quad \hat{W}^i_{i+jt} = 0 + O(\epsilon^2)\,, \tag{39}$$

where the subscript $i + j$ denotes a cross-country sum. Intermediate solutions for country-specific aggregate consumption and real wealth are then obtained as

$$\hat{C}_{it} = \frac{1}{2}\Big(\hat{C}_{i+jt} + \hat{C}_{i-jt}\Big) + O(\epsilon^2) \quad \text{and} \quad \hat{W}^i_{it} = \frac{1}{2}\hat{W}^i_{i-jt} + O(\epsilon^2)\,, \tag{40}$$

using the market clearing conditions in (39) and the intermediate solutions for cross-country differences in (38).

We obtain an intermediate solution for the real exchange rate using the goods market clearing condition in (32) together with the intermediate solutions for cross-country differenced aggregate real wealth and aggregate consumption in (38),

$$\hat{Q}_{ijt} = \eta^{(-)}_{QW} \hat{W}^i_{i-jt-1} + \eta^{(-)}_{QY} \hat{Y}_{i-jt} + \eta^{(-)}_{QV} \hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{41}$$

where the semi-partial elasticities $\eta^{(-)}_{QW}$, $\eta^{(-)}_{QY}$, and $\eta^{(-)}_{QV}$ depend on parameters of the model.

We provide detailed derivations of the intermediate solutions for cross-country differenced aggregate real wealth and consumption and for the real exchange rate in Appendix B.7.3, along with expressions for the full set of semi-partial elasticities. The solutions are intermediate because they depend on the endogenous cross-country differenced portfolio valuation effect $\hat{V}^i_{i-jt}$ defined in (35). The portfolio valuation effect has two components: cross-country differenced aggregate portfolio holdings and cross-country differenced real returns on nominal bonds. In the following section, we turn to the aggregate portfolio problem and derive solutions for these two components.

[11] We reiterate the distinction between partial and semi-partial elasticities using the example of $\mathcal{E}^{(-)}_{WW}$ and $\eta^{(-)}_{WW}$. The semi-partial elasticity $\eta^{(-)}_{WW}$ captures the indirect effect of past wealth on current wealth via consumption, but not via portfolio valuation effects. In contrast, the partial elasticity $\mathcal{E}^{(-)}_{WW}$ captures for neither indirect effect. Later, we will solve for general elasticities that capture both indirect effects.

4.2.5 The Aggregate Portfolio Problem

We now derive solutions for realized and expected returns, the portfolio valuation multiplier, and aggregate portfolio holdings. We first derive an intermediate solution for the cross-country differenced real return on nominal bonds. Using first-order Taylor expansions of the real returns defined in (8) and the quantity equations defined in (15) together with our intermediate solutions for cross-country differenced aggregate real wealth and consumption in (38), we obtain

$$\hat{R}^{i}_{i-jt} = \eta^{(-)}_{RY}\hat{Y}_{i-jt} + \eta^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + \eta^{(-)}_{RV}\hat{V}^{i}_{i-jt} + O(\epsilon^2)\,, \tag{42}$$

where $\hat{M}^{i-j}_{i-jt} \equiv \hat{M}^{i}_{it} - \hat{M}^{j}_{jt}$, and where the semi-partial elasticities $\eta^{(-)}_{RY}$, $\eta^{(-)}_{RM}$, and $\eta^{(-)}_{RV}$ depend only on the parameters of the model.

The valuation effect $\hat{V}^{i}_{i-jt}$ in (42) itself depends on the cross-country differenced real return. The expression can, therefore, be rearranged to isolate the cross-country differenced real return on the left-hand side,

$$\hat{R}^{i}_{i-jt} = \gamma^{(-)}_{RY}\hat{Y}_{i-jt} + \gamma^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + O(\epsilon^2)\,, \tag{43}$$

where $\gamma^{(-)}_{RY}$ and $\gamma^{(-)}_{RM}$ denote general elasticities. The elasticities in (43) are general in the sense that they account for general equilibrium effects of the exogenous domestic and international endowments and money supplies through all endogenous variables in the model, including portfolio valuation effects. These general elasticities are given by

$$\gamma^{(-)}_{RY} = \mu^{(-)}_{ii-ji}\eta^{(-)}_{RY} \quad \text{and} \quad \gamma^{(-)}_{RM} = \mu^{(-)}_{ii-ji}\eta^{(-)}_{RM} \quad \text{with} \quad \mu^{(-)}_{ii-ji} \equiv \frac{1}{1-\eta^{(-)}_{RV}B^{i}_{ii-ji}}\,, \tag{44}$$

where $\mu^{(-)}_{ii-ji}$ denotes the cross-country differenced portfolio valuation multiplier. The multiplier plays an important role in shaping the risk facing agents trading in bonds markets. In particular, domestic bias in aggregate portfolio holdings amplifies risk while international bias attenuates it. The following proposition, which we prove in Appendix B.8.1, emphasizes this point.

Proposition 6 (The Portfolio Valuation Multiplier)**.** *The semi-partial elasticity $\eta^{(-)}_{RV}$ is positive. Bounded domestic bias in aggregate steady-state portfolio holdings implies*

$$1 < \mu^{(-)}_{ii-ji} \quad \forall\, 0 < B^{i}_{ii-ji} < \frac{2}{\eta^{(-)}_{RV}}\,. \tag{45}$$

In this case, the multiplier amplifies the response of the cross-country differenced real return to fluctuations in endowments and money supplies. In contrast, international bias *in aggregate steady-state portfolio holdings implies*

$$0 \leq \mu^{(-)}_{ii-ji} < 1 \quad \forall\, B^i_{ii-ji} < 0. \tag{46}$$

In this case, the multiplier attenuates the response of the cross-country differenced real return to fluctuations in endowments and money supplies.

The portfolio valuation multiplier in (44) depends on cross-country differenced steady-state aggregate portfolio holdings B^i_{ii-ji}, which we have not yet determined. For this reason, (43) is not a final solution.

The first-order Taylor expansions of the aggregate equilibrium conditions used thus far leave aggregate portfolio holdings indeterminate. This indeterminacy arises because, to a first-order approximation, the bonds in our model are perfect substitutes, yielding identical expected returns. The Euler equations in (28) directly show this substitutability. Differencing across bonds, we have

$$\mathrm{E}_t\Big[R^i_{it+1}\Big] = \mathrm{E}_t\Big[R^i_{jt+1}\Big] + O(\epsilon^2)\,. \tag{47}$$

The first-order indeterminacy of portfolio holdings is a common feature of macroeconomic models with non-trivial portfolio choice, as Coeurdacier and Rey (2013) observe.

Higher-order approximation methods resolve this indeterminacy. Samuelson (1970) first developed these methods in partial equilibrium, assuming "riskiness" was "limited" and deriving order-n approximate solutions for portfolio holdings using order-$(n+2)$ approximate equilibrium conditions. Devereux and Sutherland (2011) later extended the method to general equilibrium, recognizing that Samuelson's assumption of limited risk aligns with perturbation methods used to solve dynamic stochastic general equilibrium models. We apply the Samuelson-Devereux-Sutherland method to derive aggregate portfolio holdings and later extend it to derive agent-specific portfolio holdings.

To derive order-zero approximate aggregate portfolio holdings, we require second-order approximate agent-specific Euler equations, which we then aggregate. Aggregation can be more challenging in second-order approximations because of quadratic terms, but these terms drop out when we difference Euler equations across domestic and international bonds, and the remaining terms aggregate straightforwardly. This fact allows us to state the following proposition, which we prove in Appendix B.8.2.

Proposition 7 (Second-Order Approximate Agent and Aggregate Euler Equations)**.** *Differencing second-order Taylor expansions of the agent-specific Euler equations in* (16) *across bonds, and then differencing and summing across countries yields, respectively,*

$$\begin{aligned} 0 &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i-jt+1}(\rho) - \frac{1}{\rho}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3) \quad \textit{and} \\ \frac{2}{\rho}\,\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\right] &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i+jt+1}(\rho) + \frac{1}{\rho}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,, \end{aligned} \tag{48}$$

where the inverse of the agent-specific coefficient of risk aversion multiples only aggregate variables. Multiplying by the density function in (2) *and integrating yields second-order approximate differenced and summed aggregate Euler equations, respectively,*

$$\begin{aligned} 0 &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i-jt+1} - \frac{\omega}{\bar{\rho}}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3) \quad \textit{and} \\ \omega\frac{2}{\bar{\rho}}\,\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\right] &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i+jt+1} + \frac{\omega}{\bar{\rho}}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,. \end{aligned} \tag{49}$$

The second-order approximate cross-country differenced aggregate Euler equation in (49), together with our first-order approximate solutions, suffice to determine order-zero approximate aggregate portfolio holdings. The cross-country summed version of the same, together with our first-order approximate solutions, suffice to determine second-order approximate expected cross-country differenced real returns. We now solve for these two portfolio variables.

For portfolio holdings, we substitute the first-order approximate solutions (38) for $\hat{C}_{i-jt}$, (41) for $\hat{Q}_{ijt}$, and (43) for $\hat{R}^i_{i-jt+1}$ into the second-order approximate differenced Euler equation in (49). The final solution for B^i_{ii-ji} that obtains after substitution is somewhat unwieldy; we provide it in Appendix B.8.3 and focus here on an intermediate solution in terms of cross-country differenced real returns,

$$B^i_{ii-ji} = \zeta^{(-)}_B \frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{50}$$

where $\zeta^{(-)}_B$ is a collection of model parameters, and where the conditional variance of the cross-country differenced real return and its conditional covariance with the cross-country differenced aggregate endowment are constant over time. The collection of parameters $\zeta^{(-)}_B$ is given by

$$\zeta^{(-)}_B = -\frac{\bar{\rho}\eta^{(-)}_{CY} - \omega\eta^{(-)}_{QY}}{\bar{\rho}\eta^{(-)}_{CV} - \omega\eta^{(-)}_{QV}}\,. \tag{51}$$

We use the bond market clearing condition in (32) together with the solution for cross-country differenced real bond holdings in (50) to obtain country-specific solutions,

$$B^i_{ii} = -B^i_{ij} = \frac{1}{2} B^i_{ii-ji} \,. \tag{52}$$

In Section 3, we showed empirically that aggregate portfolio holdings tend to favor domestic debt in most countries. Our theoretical model produces this bias as an endogenous outcome when agents are sufficiently risk-averse on average. The intuition for this result is straightforward: risk averse agents want to hedge their exposure to endowment fluctuations, the cross-country differenced real return $\hat{R}^i_{i-jt+1}$ covaries negatively with the cross-country differenced aggregate endowment $\hat{Y}_{i-jt+1}$, and, therefore, a long position in the domestic asset funded by a short position in the international asset produces the desired hedge. We state the sufficient condition for domestic bias in aggregate portfolio holdings in the following proposition, which we prove in Appendix B.8.3.

Proposition 8 (Domestic Bias in Aggregate Portfolio Holdings)**.** *If $\bar{\rho} > \omega$, then steady-state aggregate portfolio holdings satisfy the order-zero approximate inequalities*

$$B^i_{ij} < 0 + O(\epsilon) < B^i_{ii} \,. \tag{53}$$

For finite values of the shape parameter κ from the distribution of risk aversion, the aggregation wedge ω exceeds one. As the shape parameter approaches infinity, heterogeneity in risk aversion vanishes and the aggregation wedge ω approaches one.

The condition that $\bar{\rho}$ exceeds ω is relatively mild, and is satisfied under our assumptions on ρ_m and κ. For example, the inverse Pareto distribution shown in Figure 3 with $\kappa = 4$ implies $\omega \approx 1.33$, while Lustig and Verdelhan (2007) assume a coefficient of risk aversion that exceeds 100, and Elminejad et al. (2022) estimate a median of 3.77 and a mean of 23.36 in a 92-study meta-analysis.

Having established sufficient conditions for domestic bias in aggregate portfolio holdings, we now turn to expected returns. We obtain a second-order approximation of expected cross-country differenced real returns by substituting the first-order approximate intermediate solutions (38) for $\hat{C}_{i-jt}$ and (41) for $\hat{Q}_{ijt}$ into the second-order approximate summed Euler equation in (49). The resulting intermediate solution,

$$\begin{aligned} \mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] = {} & \frac{\omega/\bar{\rho}}{2} \, \mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} \hat{Y}_{i+jt+1}\Big] \\ & + \frac{\eta^{(-)}_{QY}}{2} \, \mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} \hat{Y}_{i-jt+1}\Big] \\ & + \frac{\eta^{(-)}_{QV}}{2} \, \mathrm{E}_t\Big[\Big(\hat{R}^i_{i-jt+1}\Big)^2\Big] + O(\epsilon^3) \,, \end{aligned} \tag{54}$$

can be reduced to a final solution using first-order approximate realized real returns in (42) and the intermediate solution for aggregate cross-country differenced portfolio holdings in (50). In the following proposition, which we prove in Appendix B.8.4, we establish sufficient conditions for the cross-country differenced expected real return to be negative.

Proposition 9 (Negative Expected Cross-Country Differenced Real Returns)**.** *If $\bar{\rho} > \omega$, then the cross-country differenced expected real return satisfies the inequality*

$$\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\right] < 0 + O(\epsilon^3) \tag{55}$$

and, to a second-order approximation, the domestic bond in country i offers a lower expected real return than the international bond after currency conversion.

Under Proposition 9, the real return on the domestic bond is lower than the real return on the international bond after accounting for movements in the real exchange rate. Intuitively, a relative rise in the domestic endowment lowers the domestic price level and raises the real return on the domestic nominal bond. At the same time, domestic agents with greater endowments import more, causing the real exchange rate to depreciate, which increases the real return on the international bond in terms of the domestic numéraire. The cross-country *differenced* real return will be negative when the depreciation effect dominates, which occurs when the preference for domestic goods is strong ($\phi > 0$) and aversion to consumption fluctuations is high ($\bar{\rho} > \omega$). Much of this intuition can be seen directly in the market clearing condition (32).

Under Propositions 8 and 9, country i holds an aggregate long position in the domestic bond and an aggregate short position in the international bond, leading to an expected negative real return. Thus, aggregate portfolios exhibit domestic bias despite low domestic interest rates. We now focus on individual agents to establish conditions for carry traders in such economies. We will divide the solution into non-portfolio and portfolio components, paralleling our approach for the aggregate economy.

4.2.6 The Agent Non-Portfolio Problem

Agent-specific and aggregate equilibrium conditions are structurally similar, so we solve for agent-specific variables in deviations from aggregate variables rather than from the steady state. This approach not only provides analytical convenience but also strengthens the link between individual and aggregate solutions.

Our solution procedure follows the one used for aggregate variables, but for individual agents we must solve systems of equations in both cross-country differences and sums, because we cannot rely on market clearing conditions as we did in the aggregate problem. Once we have solutions for these differences and sums, we can derive country-specific solutions for agents' consumption, real wealth, and portfolio holdings.

Our solutions depend on two state variables: agent-specific and aggregate real wealth. Since each agent is measure zero, these variables evolve independently. The presence of aggregate real wealth as a second state variable does complicate the problem, but only slightly because agents treat it as exogenous in their decision-making.

We begin with the cross-country differenced system for individual agents. Because the steps to solve the cross-country summed system follow the same logic, we present the cross-country summed system in Appendix B.9.4 to save space.

Cross-Country Differences. We derive the cross-country differenced system for an individual agent from the first-order approximate individual and aggregate Euler equations in (28) and (29) and budget constraints in (30). We first take cross-country differences for the individual and aggregate equilibrium conditions separately, then take the difference between the individual and aggregate cross-country differences.

We obtain the first-order approximate cross-country differenced agent-specific Euler equation in deviations from aggregates,

$$\begin{aligned}&\Big(\hat{C}_{i-jt}(\rho)-\hat{C}_{i-jt}\Big)-\theta\Big(\hat{W}^i_{i-jt}(\rho)-\hat{W}^i_{i-jt}\Big)\\&\quad=\beta R\,\mathrm{E}_t\left[\Big(\hat{C}_{i-jt+1}(\rho)-\hat{C}_{i-jt+1}\Big)-\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\Big(\hat{Q}_{ijt+1}-\hat{Q}_{ijt}\Big)\right]+O(\epsilon^2)\,,\end{aligned}\tag{56}$$

where the subscript $i-j$ on agent-specific variables denotes cross-country differences between agents with identical coefficients of relative risk aversion, and where the intermediate solution for the real exchange rate in (41) can be used to eliminate the real exchange rate. Similarly, we obtain the first-order approximate agent-specific budget constraint in deviations from aggregates,

$$\begin{aligned}\Big(\hat{W}^i_{i-jt}(\rho)-\hat{W}^i_{i-jt}\Big)&=R\Big(\hat{W}^i_{i-jt-1}(\rho)-\hat{W}^i_{i-jt-1}\Big)\\&\quad-\Big(\hat{C}_{i-jt}(\rho)-\hat{C}_{i-jt}\Big)+R\Big(\hat{V}^i_{i-jt}(\rho)-\hat{V}^i_{i-jt}\Big)+O(\epsilon^2)\,,\end{aligned}\tag{57}$$

where we introduce the cross-country differenced agent portfolio valuation effect $\hat{V}^i_{i-jt}(\rho)$, defined as

$$\hat{V}^i_{i-jt}(\rho)\equiv B^i_{ii-ji}(\rho)\hat{R}^i_{i-jt}\quad\text{with}\quad B^i_{ii-ji}(\rho)\equiv B^i_{ii}(\rho)-B^i_{ji}(\rho)\,.\tag{58}$$

In the Euler equation (56) and the budget constraint (57), the agent's consumption always appears as a deviation from aggregate consumption, which never enters separately. Therefore, we can treat the deviation of an individual's consumption from the aggregate as a single variable. Similarly, the agent's real wealth appears as a deviation from aggregate real wealth, but aggregate real wealth also affects the equations separately through the real exchange rate in (41). As a result, we must treat individual and aggregate real wealth as separate state variables and include the law of motion for aggregate real wealth (38) in the agent's cross-country differenced system.

The agent's cross-country differenced system is then given by three simultaneous difference equations: the Euler equation in (56), the budget constraint in (57), and the law of motion for aggregate real wealth in (38). After eliminating the real exchange rate using the intermediate solution in (41), we write the system in matrix form as

$$\begin{aligned}\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big] &= \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} \\ &\quad + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}\hat{V}^i_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}\hat{V}^i_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{59}$$

where $\hat{\boldsymbol{Z}}_{i-jt}(\rho) = \Big[\hat{W}^i_{i-jt-1}(\rho) \quad \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt} \quad \hat{W}^i_{i-jt-1}\Big]'$ denotes the (3×1) vector of the agent's cross-country differenced real wealth, cross-country differenced consumption in deviation from aggregate, and cross-country differenced aggregate real wealth, and where $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}$, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}$, and $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}$ denote matrices of partial elasticities. For example, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ is (3×3) with elements $\mathcal{E}^{(-)}_{mn}$, $m, n \in \{W(\rho), C(\rho), W\}$. We provide a derivation of the agent's cross-country differenced system (59) in Appendix B.9.1, along with expressions for the full set of partial elasticities.

In Proposition B1 in Appendix B.9.2, we establish uniqueness and stationarity for the agent cross-country differenced system. There, we follow the same strategy as we did with the aggregate cross-country differenced system, considering a neighborhood of small positive values for θ near the knife-edge non-stationary value of $\theta = 0$ to establish the result.

As we did for the aggregate system, we proceed under the assumption that θ takes a small positive value, and derive an intermediate solution to the agent's cross-country differenced system in (59) by factorizing the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ into a diagonal matrix of eigenvalues and a matrix of eigenvectors and by imposing the condition that the endogenous variables in the system do not "explode too fast," again as in Blanchard and Kahn (1980). Our intermediate solutions to the agent's non-portfolio problem depend on the agent's lagged real wealth in deviation from aggregate, lagged aggregate real wealth independently, endowments, and portfolio valuation effects. Later, once the agent's portfolio problem has been solved, these intermediate non-portfolio solutions will combine with portfolio solutions to yield general solutions.

We introduce the "vanishing integral property" for deviations of agent variables from aggregate variables, denoting such deviations with tildes. Aggregating these variables across agents results in zero because the deviations average out. These variables remain specific to individual agents and retain the index ρ. For real wealth, consumption, and portfolio valuation effects, the deviations are expressed as

$$\begin{gathered}\tilde{W}^i_{i-jt}(\rho) = \hat{W}^i_{i-jt}(\rho) - \hat{W}^i_{i-jt}\,, \quad \tilde{C}_{i-jt}(\rho) = \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt}\,, \\ \text{and} \quad \tilde{V}^i_{i-jt}(\rho) = \hat{V}^i_{i-jt}(\rho) - \hat{V}^i_{i-jt}\,.\end{gathered} \tag{60}$$

Some semi-partial elasticities in our intermediate solutions below will also have this property, and we also denote these with tildes. All semi-partial elasticities without tildes are independent of agent-specific coefficients of relative risk aversion and therefore common to all agents.

With this notation in place, we write our intermediate solution for the agent's cross-country differenced system in deviations from aggregates as

$$\begin{aligned}\begin{bmatrix}\tilde{W}^i_{i-jt}(\rho)\\ \tilde{C}_{i-jt}(\rho)\end{bmatrix} &= \boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)}\tilde{W}^i_{i-jt-1}(\rho) + \boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)}\tilde{V}^i_{i-jt}(\rho)\\ &\quad + \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V}\hat{V}^i_{i-jt} + \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{61}$$

where $\boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)}$, $\boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)}$, $\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V}$, and $\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y}$ denote matrices of semi-partial elasticities. The tilde notation emphasizes the consistency between agent and aggregate problems, as aggregating 61 across agents results in zero. We derive this intermediate solution and provide expressions for the full set of semi-partial elasticities in Appendix B.9.3.

Cross-Country Sums. We solve the cross-country summed system using the same method as for the cross-country differenced system. The summed system is simpler because it does not involve the real exchange rate and has only one predetermined state variable, agent-specific real wealth.

For brevity, we present solution in Appendix B.9.6. There, we establish stationarity and uniqueness in Proposition B2, before solving for cross-country summed agent consumption and real wealth in deviations from aggregates. The solution depends on lagged real wealth, lagged endowments, and portfolio valuation effects. Together, the cross-country differenced and summed solutions yield country-specific solutions for agent consumption and real wealth.

4.2.7 The Agent Portfolio Problem

In our model, agents endogenously choose a continuum of portfolio positions according to their heterogeneous risk preferences. Depending on these preferences, agents become hedgers, speculators, or hand-to-mouth consumers. Relatively risk-tolerant agents opt for speculative carry trade positions.

We establish this result by extending the Samuelson-Devereux-Sutherland method of higher-order approximation to our heterogeneous-agent setting. This extension allows us to derive closed-form solutions for the portfolio positions of individual agents in a general equilibrium continuum economy. Intuitively, agents with higher risk aversion prefer portfolios with less volatility and lower average returns, leading to consumption that is less volatile and lower on average. In contrast, agents with lower risk aversion choose portfolios with more volatility and higher average returns, resulting in consumption that is more volatile and higher on average. Given the range of different risk preferences, this model generates rich heterogeneity in investment and consumption patterns across agents within each country.

The portfolio problem of individual agents differs from the aggregate portfolio problem because there are no bond market clearing conditions for individual agents, unlike for the aggregate economy. Instead, we use the second-order approximate cross-country differenced and summed agent Euler equations in (48) to derive order-zero approximate cross-country differenced and summed agent portfolio holdings, respectively. We then combine these results to obtain country-specific solutions.

By differencing the second-order approximate cross-country differenced agent and aggregate Euler equations in (48) and (49), we obtain an Euler equation with cross-country differenced agent-specific consumption expressed in deviation from aggregate,

$$0 = \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\tilde{C}_{i-jt+1}(\rho) - \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,. \tag{62}$$

Combining (62) with our intermediate solution in (61) for agent-specific cross-country differenced consumption and the intermediate solution for the real exchange rate in (41), we obtain an intermediate order-zero approximate solution for agent-specific cross-country differenced holdings of domestic bonds in deviation from aggregate,

$$\tilde{B}^i_{ii-ji}(\rho) = \zeta^{(-)}_{B(\rho)}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{63}$$

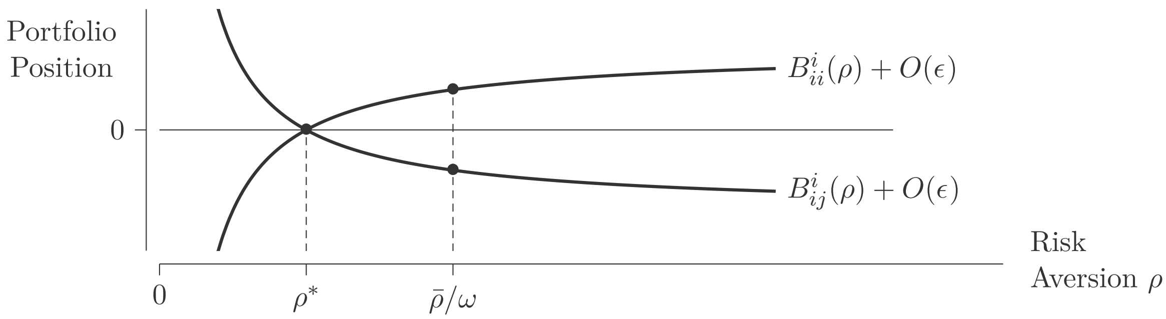
where $\tilde{B}^i_{ii-ji}(\rho) \equiv B^i_{ii-ji}(\rho) - B^i_{ii-ji}$, and where the variance of the cross-country differenced real return and its covariance with the cross-country differenced endowment are constant. Here, $\zeta^{(-)}_{B(\rho)}$ is a collection of model parameters independent of any individual agent's coefficient of relative risk aversion. We provide an expression for $\zeta^{(-)}_{B(\rho)}$ in Appendix B.10.

By differencing the second-order approximate cross-country summed agent and aggregate Euler equations (48) and (49), we obtain an Euler equation with cross-country summed agent-specific consumption expressed in deviation from aggregate,

$$0 = \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\tilde{C}_{i+jt+1}(\rho) - \frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Y}_{i+jt+1}\right)\right] + O(\epsilon^3)\,. \tag{64}$$

Combining (64) with the intermediate solution for cross-country summed agent-specific consumption in appendix equation (B113), we obtain an intermediate order-zero approxi-

Figure 4 – Cross-Section of Agent Portfolio Holdings



Notes. The figure shows portfolio holdings of the domestic bond $B^i_{ii}(\rho)$ and international bond $B^i_{ij}(\rho)$ for agents in country i. The horizontal axis shows agents' coefficients of relative risk aversion and the vertical axis shows agents' portfolio positions. Agents' domestic bond positions rise in risk aversion, while their international bond positions fall. The agent with $\rho = \rho^*$ is hand-to-mouth. Agents with $\rho < \rho^*$ are carry traders with short domestic bond positions and long international bond positions. The agent with $\rho = \bar{\rho}/\omega$ holds a portfolio with domestic bias that matches the aggregate portfolio.

mate solution for agent-specific cross-country summed bond holdings,

$$\tilde{B}^i_{ii+ji}(\rho) = \zeta^{(+)}_{B(\rho)} \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}} \right) \frac{\mathrm{E}_t \left[\hat{R}^i_{i-jt+1} \hat{Y}_{i+jt+1} \right]}{\mathrm{E}_t \left[\left(\hat{R}^i_{i-jt+1} \right)^2 \right]} + O(\epsilon) \,, \tag{65}$$

where $\tilde{B}^i_{ii+ji}(\rho) \equiv B^i_{ii+ji}(\rho) - B^i_{ii+ji}$, and where the variance of the cross-country differenced real return and its covariance with the cross-country summed endowment are constant. Here, $\zeta^{(+)}_{B(\rho)}$ is a collection of model parameters independent of agent-specific coefficients of relative risk aversion. We provide an expression for $\zeta^{(+)}_{B(\rho)}$ in Appendix B.10.

Combining agent-specific cross-country differenced and summed domestic bond holdings, we obtain an intermediate country-specific solution,

$$\tilde{B}^i_{ii}(\rho) = \frac{1}{2} \left(\tilde{B}^i_{ii-ji}(\rho) + \tilde{B}^i_{ii+ji}(\rho) \right) + O(\epsilon) \,, \tag{66}$$

where $\tilde{B}^i_{ii}(\rho) \equiv B^i_{ii}(\rho) - B^i_{ii}$, and where $B^i_{ii}(\rho) = -B^i_{ij}(\rho)$ from Proposition 2. A final solution for $\tilde{B}^i_{ii}(\rho)$ can be obtained directly from (66) using (42), (63), and (65).

Economies with domestic bias in aggregate portfolios earning low returns are not engaged in aggregate carry trades. To model carry trades in such economies, individual carry traders must be outweighed by agents who favor domestic assets. Our heterogeneous-agent model captures this cross-sectional pattern of investment, as we establish in the following culminating proposition, which we prove in Appendix B.10.1.

Proposition 10 (International Bias in Agent-Specific Portfolio Holdings)**.** *If $\bar{\rho} > \omega$, then* *a threshold level of risk aversion* $\rho^* \in (0, \bar{\rho}/\omega)$ *exists such that*

$$\begin{aligned} B^i_{ij} < 0 + O(\epsilon) < B^i_{ii} \quad \textit{while} \\ B^i_{ii}(\rho) < 0 + O(\epsilon) < B^i_{ij}(\rho) \quad \forall \rho < \rho^* \,. \end{aligned} \tag{67}$$

To an order-zero approximation, the aggregate portfolio position in country i *exhibits domestic bias while individual portfolio positions exhibit international bias for agents with coefficients of relative risk aversion below the threshold value* ρ^**.*

Figure 4 illustrates order-zero approximate holdings of domestic and international bonds for the cross-section of individual agents in country i. In the illustration, country i's aggregate portfolio is biased towards the domestic asset. The agent with $\rho = \rho^*$ is hand-to-mouth, agents with $\rho < \rho^*$ hold carry trade portfolios earning positive expected returns, and agents with $\rho > \rho^*$ hold portfolios with domestic bias earning negative expected returns. The agent with $\rho = \bar{\rho}/\omega$ holds exactly the aggregate portfolio. In this way, the model endogenizes a continuum of investor types within a tractable, two-country, dynamic stochastic general equilibrium framework.

5 Conclusions

The currency carry trade literature has focused on rationalizing returns without addressing portfolio positions, yet realizing carry trade returns requires holding carry trade portfolios. We document strong domestic bias in aggregate debt holdings by country and currency. For low-interest-rate economies like Germany, Japan, and the United States, this domestic bias does not resemble an aggregate carry trade. Individual carry traders must therefore be outweighed by agents favoring domestic-currency assets. A complete theory should explain both carry trade returns and positions, while acknowledging that most agents are not carry traders—a fact representative-agent models cannot accommodate.

Our empirical findings highlight a tension between the finance literature on the carry trade and the international macroeconomics literature on domestic bias in aggregate portfolios. We resolve this tension using a theoretical model that features heterogeneity in risk aversion and captures a full range of individual portfolio positions. To solve the model, we extend the Samuelson-Devereux-Sutherland solution method of higher-order approximations to heterogeneous agents. The model is determinate, stationary, and highly tractable, admitting closed-form solutions for both individual and aggregate variables in a familiar discrete-time setting, producing rich heterogeneity in investment and consumption behavior. Individual portfolio positions in the model endogenously range from speculative carry trades, to hand-to-mouth positions, to domestically-biased hedges, while the aggregate portfolio displays domestic bias. The model reconciles the existence of individual carry traders with the empirical observation of domestic bias in aggregate portfolios that earn low returns.

At the core of the model is a trade-off between the hedge that portfolios with domestic bias provide and the lower returns they offer. When the home endowment rises relative to foreign, the price of the home good falls and the real return on the home nominal bond rises. Home agents consume more of both home and foreign goods, boosting imports and depreciating the home real exchange rate. Consequently, the real return on the foreign nominal bond also rises for home agents. The *difference* in real returns—home minus foreign—depends on the responsiveness of the real exchange rate, which is greater when agents have higher average risk aversion and a stronger preference for the domestic good. The real exchange rate response dominates in our model, and the foreign real return rises above home, rendering long positions in the home bond combined with short positions in the foreign bond an effective hedge against consumption growth risk. Agents with higher risk aversion take this hedge, accepting a negative difference in returns, while agents with lower risk aversion take carry trades to exploit a positive difference in returns.

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