Online Appendix

Who Carries?*

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Abstract

This online appendix provides additional materials to support the main paper, and is divided into an empirical and a theoretical section. The empirical section describes our empirical methodology and its limitations, provides further empirical analysis, and presents robustness checks. The theoretical section provides derivations of our heterogeneous-agent model of the carry trade, and proves the propositions presented in the paper.

JEL Codes: F31, F41, G15

Keywords: Carry trade, home bias, risk aversion, portfolio choice, heterogeneous agents

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A Empirical Appendix

Appendix A details the empirical methodology used in Section 3. It outlines the reporting standards for short positions (A.1), describes the procedures for estimating bilateral debt holdings by issuer and currency (A.2), and explains the computation of domestic currency bias and portfolio returns (A.3).

A.1 Short Position Reporting Standards

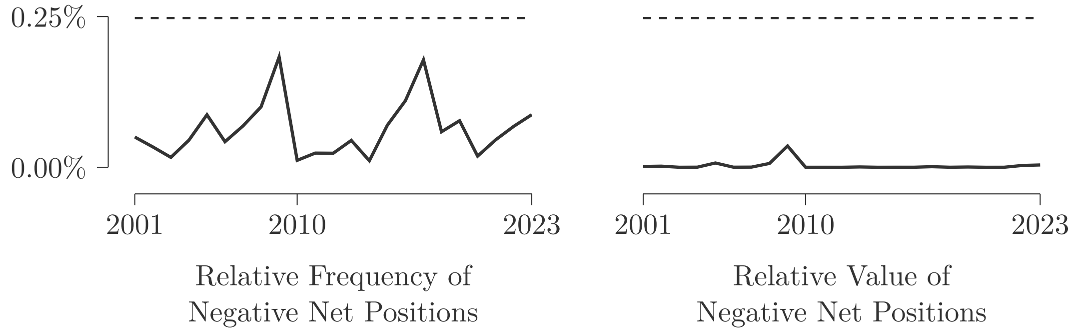
The main IMF reporting standards for international financial statistics are published in two documents: the Balance of Payments Manual (BPM) and the CPIS Survey Guide (CSG). The BPM is the more established of the two standards, it was first published in 1948, and it governs broadly the reporting of international financial statistics at the country level. The CSG is narrower in scope, and younger than the BPM. Until the turn of the century, no definitive reporting standard for short positions appeared in either publication: the fifth edition BPM makes no mention of short positions whatsoever (International Monetary Fund, 1993), and the first edition CSG describes possible methods for reporting short positions, but provides no definitive standard (International Monetary Fund, 1996, para. 93 and Box 4). After the turn of the century, the second edition CSG published clearer guidance: "If (when) the security is on-sold, the 'borrower' of the security should record a 'short' position" (International Monetary Fund, 2002, para. 3.78), and then, seven years later, the sixth edition BPM established the definitive standard that applies today:

> Short positions occur when an institutional unit sells securities for which it is not the economic owner. [...] Delivery to the purchaser is made through the use of a borrowed security. The party with the short position records a negative value for the holding of the asset. The short position is shown as a negative asset, rather than a liability. (International Monetary Fund, 2009, para. 7.28)

The most recent third edition CSG now cites the sixth edition BPM when describing short position reporting (Josyula, 2018, para. 3.52). A third document, the Handbook on Securities Statistics (HSS), jointly produced by the IMF, the BIS, and the ECB, was released in parts starting in 2009. Its second part, published in 2010, covers debt securities holdings and follows the sixth edition BPM (International Monetary Fund, 2010, para 4.15).

While 2009 marks the introduction of a definitive standard for the reporting of short positions in international financial statistics, the reporting standard was not immediately implemented by all reporting countries. Press releases and country-level reporting guides from the years following the publication of the sixth edition BPM indicate that the

Figure A1 – Negative Net Positions in International Debt Holdings



Notes. The figures show the relative count and value of reported negative net debt positions, aggregated across CPIS-reporting countries annually. Aggregate net positions equal the sum of gross long and gross short positions, with the latter being negative or zero. The left panel shows the relative frequency of negative net positions, calculated as their count divided by the count of all net positions. The right panel shows the relative value of negative net positions, computed as their absolute value divided by the value of all net positions.

standard had been implemented by many countries by 2015.[1] For example, the ECB announced that Eurostat would disseminate statistics under the new standard beginning in October 2014 (European Central Bank, 2014a). Figure A1 shows no significant change in relative counts and relative values of reported short positions in the years following the publication of the new reporting standard in 2009, and values remain low.

A.2 Estimating Debt Holdings by Issuer and Currency

This section describes how we estimate country-level debt holdings by currency, which we use in Table 1 and Figure 2. Our approach combines the procedures in Fidora et al. (2007) for estimating investor-country holdings of domestic debt and in Lane and Shambaugh (2010) for estimating the currency composition of investor-country holdings of international debt.[2]

Throughout this section, *debt holdings* refer to country-level aggregate net debt holdings in nominal terms in numéraire currency USD, where aggregate net means the sum of long positions (with positive sign) and short positions (with negative sign) over all agents (households, firms, government) residing in a given country. Abusing the notation of our theoretical model, we denote these holdings B_{ijt}, although they are nominal here. Domestic debt holdings refer to debt held by residents of a given country that was issued by residents of the same country. International debt holdings refer to debt held by residents of a given country that was issued by residents of another country. Debt may be issued in

[1] The United States has not adopted the BPM6 standard for reporting short positions, instead reporting gross long positions to CPIS without netting gross short positions.

[2] Allen et al. (2023) update the work of Lane and Shambaugh (2010) using confidential IMF survey data. However, their estimates exclude domestic holdings and are available by currency only, rather than by currency and issuer, making them less suitable for our study.

domestic or international markets (as defined by the BIS), and the issuance market, as a concept, is distinct from the residence of the issuer or holder of debt.[3]

In subsection A.2.1, we use gravity models to estimate the domestic and international debt holdings of investor countries by issuer country. We first estimate a model for international debt holdings, then a second model for both domestic and international debt holdings. In subsection A.2.2, we estimate currency weights, which allow us to compute domestic and international debt holdings of investor countries, by issuer country *and* issuance currency. The initial weights are based on the reported currency composition of debt issued in international markets, but we adjust these weights to account for holdings of international debt issued in the domestic market of the issuer. Finally, we use a biproportional fitting procedure to reconcile the estimated debt holdings of investor country by issuer country and issuance currency with reported debt holdings by and issuance currency alone.

A.2.1 Estimating Debt Holdings by Issuer Country

We now estimate portfolio holdings by issuer country. We proceed in three steps. In step one, we fit a first-stage gravity model to CPIS-reported international debt holdings, and use the fitted first-stage model to obtain first-stage predictions of international debt holdings for non-reporting countries. In step two, we estimate domestic debt holdings as the residual difference between total debt issuance and rest-of-world international debt holdings for each issuer country. In step three, we fit a second-stage gravity model to domestic and international debt holdings, and use the fitted second-stage model to obtain final predictions of domestic and international debt holdings for non-reporting countries.

Step One. We fit a first-stage gravity model to CPIS-reported international debt holdings using pseudo-Poisson maximum likelihood (PPML), which performs well in the presence of heteroskedasticity (Silva and Tenreyro, 2006; Santos Silva and Tenreyro, 2022). Our unbalanced panel contains 177 981 investor-issuer-year observations spanning 2001–2021 and covering 83 investor countries and 196 issuer countries. Because we use the model to predict missing debt holdings for non-reporting countries, we limit regressors to those with good coverage across countries to minimize data loss.

The gravity variables we select, obtained from Conte and Mayer (2022), include geographic distance, a contiguity indicator, a common language indicator, GDP, GDP per capita, an EU membership indicator, and a WTO membership indicator. Additionally, we construct a common-currency indicator, a tax haven indicator based on the list in

[3]For example, a German resident might issue German debt in German debt markets under German law, that is held by a US resident. From the US perspective, this represents holdings of international debt issued in the German domestic market. Alternatively, a German resident might issue German debt in a US market under US law, that is held by a German resident. From the German perspective, this represents holdings of domestic debt issued in the German international market.

Coppola et al. (2021) (used for interaction terms), and a crisis indicator for both investor and issuer countries from the banking, debt, and currency crises identified by Nguyen et al. (2022). We also include interactions for investor and issuer EU membership, WTO membership, GDP, GDP per capita. Finally, we include investor, issuer, and year fixed effects. Our specification is given by

$$\mathrm{E}\Big[B_{ijt} \mid \boldsymbol{X}_{ijt},\, \boldsymbol{IT}_{ijt},\, \boldsymbol{FE}_{ijt}\Big] = \exp\Big(\boldsymbol{\beta}_X' \boldsymbol{X}_{ijt} + \boldsymbol{\beta}_{IT}' \boldsymbol{IT}_{ijt} + \boldsymbol{\beta}_{FE}' \boldsymbol{FE}_{ijt}\Big)\,, \quad i \neq j\,, \qquad \text{(A1)}$$

where B_{ijt} denotes investor country i's holding of debt issued by country j in year t. The vector $\boldsymbol{X}_{ijt}$ collects a constant and the gravity and indicator variables. The vector $\boldsymbol{IT}_{ijt}$ collects interaction terms, and $\boldsymbol{FE}_{ijt}$ collects fixed effects.

After dropping 5 multicollinear fixed effects using QR decomposition, we estimate the first-stage model with 302 regressors and 171 789 observations.[4] We estimate the model both with and without L2 regularization ($\alpha = 0.0001$). Our preferred estimates, used in Table 1 and Figure 2, include regularization. First-stage results for a subset of regressors are reported in columns two and four of Table A1. The correlation between predicted and observed values is 93.0 without regularization and 92.88 with regularization.

Step Two. We compute domestic holdings of domestic debt as the residual between total reported issuance amounts outstanding and the sum of reported plus gravity-estimated rest-of-world holdings for issuer countries with available data.

Total issuance amounts outstanding are available for 38 countries in the BIS Debt Securities Statistics (DSS) with good coverage over our sample period. For 20 additional countries with poor or no DSS coverage, we approximate total issuance by summing domestic and international market issuance amounts from the BIS International Debt Statistics (IDS).[5]

To measure rest-of-world holdings of issuer-country debt, we aggregate CPIS-reported holdings for reporting countries and first-stage gravity-estimated holdings for non-reporting countries. This aggregate covers all economic sectors (including government) except for central banks. Central Bank holdings are reported separately in the IMF's Securities Held as Foreign Exchange Reserves (SEFER) survey. We add SEFER-reported holdings to the CPIS aggregate to obtain an estimate of total rest-of-world holdings.

Following Fidora et al. (2007), we define domestic holdings of domestic debt as the

[4] Our implementation of QR decomposition removes the fixed effects with the lowest-valued diagonal elements of the R matrix until the condition number equals approximately 50.

[5] As Gruić and Wooldridge (2012) note, summing domestic and international issuance may involve double-counting. However, in cases where both DSS and IDS total issuance are available, we find the totals to be very similar. DSS data represent 87.13% of total reported issuance in an average year in our sample, and IDS data represent 12.87%.

Table A1 – First- and Second-Stage PPML Gravity Model Estimation

Independent Variable	No Regularization		L2 Regularization	
	Stage One	Stage Two	Stage One	Stage Two
Constant	−17.91	−11.95	−7.07	−7.62
	(0.00)	(0.00)		
Distance	−0.39	−0.48	−0.38	−0.48
	(0.00)	(0.00)		
Common Language	0.23	0.27	0.23	0.27
	(0.00)	(0.00)		
Common Currency	0.68	0.93	0.68	0.93
	(0.00)	(0.00)		
Investor GDP	3.50	1.75	1.22	1.23
	(0.00)	(0.00)		
Issuer GDP	4.71	1.15	1.86	1.76
	(0.00)	(0.00)		
Investor GDP/Cap	−0.68	1.26	1.31	1.67
	(0.00)	(0.00)		
Issuer GDP/Cap	−1.81	1.76	0.67	1.27
	(0.00)	(0.00)		
Domestic Debt		6.19		6.14
		(0.00)		
Domestic Debt × Investor GDP		−0.49		−0.48
		(0.00)		
Domestic Debt × Investor GDP/Cap		−2.69		−2.67
		(0.00)		
Domestic Debt × Investor Tax Haven		−1.42		−1.40
		(0.00)		
Year FE	Yes	Yes	Yes	Yes
Investor and Issuer FE	Yes	Yes	Yes	Yes
Observations	171 789	172 811	171 789	172 811
Correlation	93.00	99.55	92.88	99.55

Notes. The table reports PPML estimates from the first- and second-stage gravity models described in Section A.2.1. The first stage uses CPIS-reported international debt holdings; the second stage incorporates domestic debt holdings computed as the residual between total issuance and rest-of-world holdings. P-values appear below coefficient estimates, and the observation counts and correlation between observed and predicted holdings are reported at the bottom of the table. Columns labeled "L2 Regularization" apply a penalty parameter of $\alpha = 0.0001$, and no p-values are available for those specifications. Fixed effects and less economically-significant covariates are omitted.

difference between total issuance amounts outstanding and total rest-of-world holdings:

$$B_{iit} = I_{it} - B_{-iit}\,, \quad \text{where } B_{-iit} \equiv \sum_{j \neq i} B_{jit} + B_{CBit}\,, \tag{A2}$$

where I_{it} denotes issuer-country i's total debt issuance amounts outstanding, B_{CBit} represents SEFER-reported central bank holdings of country i's debt, and B_{-iit} denotes rest-of-world holdings of debt issued by country i.

For six countries (Estonia, India, Ireland, Lebanon, Luxembourg, and Taiwan), CPIS-reported rest-of-world holdings alone, before adding gravity-estimated holdings, exceed total reported outstanding issuance. In these cases, which together account for 2.27% of total reported outstanding issuance in an average year, the residual-based estimates of domestic debt holdings become negative and we exclude them, instead relying on second-stage gravity estimates, described next.

Step Three. We fit a second-stage gravity model that incorporates both the CPIS-reported international debt holdings and the residual-based domestic debt holdings from the previous steps. We continue to use the PPML estimator and the same baseline regressors, but we now add a domestic debt indicator (equaling one if the investor and issuer countries coincide) and interact this indicator with GDP, GDP per capita, and the tax haven indicator. All continuous variables remain logged and standardized. Our specification is given by

$$\begin{aligned} &\mathrm{E}\Big[B_{ijt} \mid \boldsymbol{X}_{ijt},\, \boldsymbol{IT}_{ijt},\, \boldsymbol{FE}_{ijt},\, \boldsymbol{DOM}_{ijt}\Big] \\ &\quad = \exp\Big(\boldsymbol{\beta}_X^\top \boldsymbol{X}_{ijt} + \boldsymbol{\beta}_{IT}^\top \boldsymbol{IT}_{ijt} + \boldsymbol{\beta}_{FE}^\top \boldsymbol{FE}_{ijt} + \boldsymbol{\beta}_{DOM}^\top \boldsymbol{DOM}_{ijt}\Big)\,, \end{aligned} \tag{A3}$$

where B_{ijt} now encompasses both reported international and residual-based domestic debt holdings. The vector $\boldsymbol{DOM}_{ijt}$ contains a domestic debt indicator that equals one for $i = j$ and zero otherwise, and its interactions.

Columns three and five of Table A1 present the second-stage results. The coefficient on the domestic debt indicator is economically and statistically significant, reflecting a strong domestic bias. By contrast, the interaction terms with GDP per capita and the tax haven indicator are significantly negative, suggesting that wealthier countries and tax havens exhibit a smaller *incremental* domestic bias once we control for other factors. This does not imply that they hold less domestic debt in absolute terms or as a share of total debt; rather, it implies only that the domestic debt indicator effect is attenuated.

After fitting the second-stage model, we recalculate rest-of-world holdings by combining CPIS-reported debt holdings with second-stage gravity-estimated debt holdings for non-reporting countries, then update our residual-based estimates of domestic debt holdings accordingly. Our final data set of bilateral debt holdings thus includes both reported and estimated holdings of international and domestic debt for all countries in our sample.

A.2.2 Estimating Weights by Issuer Country and Issuance Currency

To obtain estimates of investor-country debt holdings by issuer and currency, we proceed in five steps. First, we compute *holding weights* to capture each issuer's share in the total holdings of each investor country. Second, we compute international-market *issuance weights* to capture the currency denomination of debt issued by each investor country in international markets. Third, we compute *access weights* to capture the extent to which domestic and international investors access the issuer's domestic and international markets. Fourth, we combine these issuance and access weights to obtain initial *portfolio weights*. Fifth, we adjust the initial portfolio weights using a biproportional fitting algorithm to match the currency composition of total debt holdings reported by investor countries in CPIS. The final portfolio weights decompose each investor country's total debt holdings into holdings by issuer country and issuance currency.

Step One: Holding Weights. We compute holding weights as issuer country shares in the total debt holdings of investor countries. Holding weights for investor-country i satisfy following identity,

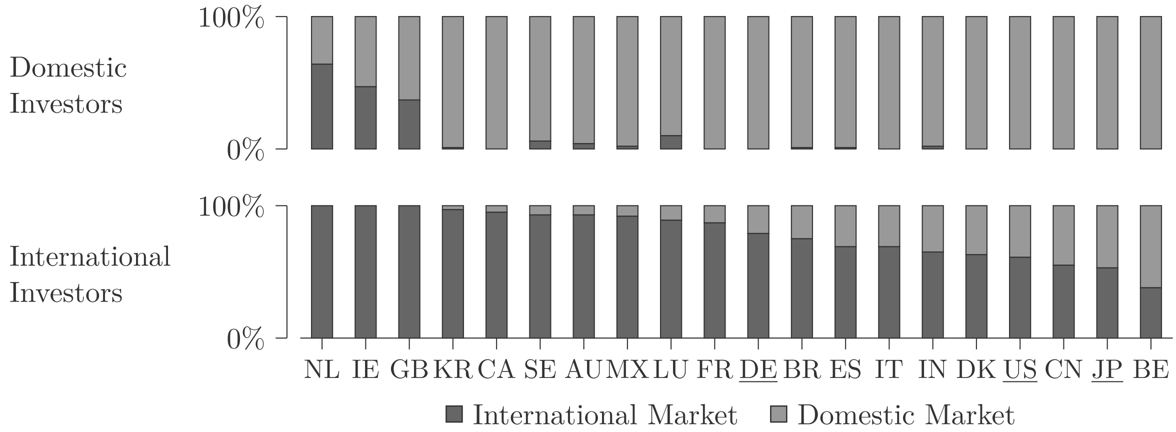
$$B_{it} = \sum_j B_{ijt} \quad \Leftrightarrow \quad 1 = \sum_j b_{ijt}\,, \tag{A4}$$

where B_{it} denotes the total debt holdings of country i and $b_{ijt} \equiv B_{ijt}/B_{it}$ denotes the share of debt issued by country j in the total debt holdings of country i.

Step Two: Access Weights. We compute access weights based on the difference between total rest-of-world holdings and total international-market debt issuance for each issuer country. If the difference is positive, with rest-of-world holdings exceeding international-market issuance, we infer that international investors hold debt from the issuer's domestic market. If the difference is negative, with international-market debt issuance exceeding rest-of-world holdings, we infer that domestic investors hold debt from the issuer's international market.[6]

[6]The BIS distinguishes between debt issued in domestic and international markets. In domestic markets, issuers predominantly issue debt in their domestic currency. Although international-market debt also is increasingly denominated in the issuer's domestic currency (see Gruić and Wooldridge 2012, Du and Schreger 2016, and Burger et al. 2018), a substantial share remains in USD or other international currencies. Because the currency composition differs by market, the market accessed by investors matters for our estimates. Gruić and Wooldridge (2012) document a divergence between international bond issues (from BIS IDS data) and cross-border bond holdings (from IMF CPIS data). This discrepancy suggests that international investors sometimes buy debt in the issuer's domestic market, while domestic investors sometimes buy debt in international market of their own country. Burger and Warnock (2007) study this phenomenon in the bond markets of 40 countries.

Figure A2 – Market Access Weights for Top-Twenty Issuer Countries



Notes. The figure shows access weights for domestic and international investors in the top 20 issuer countries by total issuance, averaged over the period 2001–2021. The upper panel shows weights for domestic investors, who generally buy domestic debt from domestic markets. The lower panel shows weights for international investors, who generally buy international debt from international markets. In 5.87% of country-year observations, weights fall outside the interval $[0, 1]$ and are truncated to zero or one.

We compute separate access weights for domestic and international investors. For domestic investors residing in issuer country i, access weights satisfy the following identity,

$$\begin{aligned} B_{iit} &= \max\Big(I_{it}^{(IM)} - B_{-iit}, 0\Big) + \Big(B_{iit} - \max\Big(I_{it}^{(IM)} - B_{-iit}, 0\Big)\Big) \\ \Leftrightarrow \quad 1 &= a_{iit}^{(IM)} + a_{iit}^{(DM)}, \end{aligned} \tag{A5}$$

where $a_{iit}^{(IM)} \equiv \max(I_{it}^{(IM)} - B_{-iit}, 0)/B_{iit}$ denotes the domestic-investor access weight to issuer i's international market, and $a_{iit}^{(DM)} = 1 - a_{iit}^{(IM)}$ denotes the domestic-investor access weight to issuer i's international market. For international investors outside issuer country i, access weights satisfy a similar identity,

$$\begin{aligned} B_{-iit} &= \max\Big(B_{-iit} - I_{it}^{(IM)}, 0\Big) + \Big(B_{-iit} - \max\Big(B_{-iit} - I_{it}^{(IM)}, 0\Big)\Big) \\ \Leftrightarrow \quad 1 &= a_{-iit}^{(DM)} + a_{-iit}^{(IM)}, \end{aligned} \tag{A6}$$

where $a_{-iit}^{(DM)} \equiv \max(B_{-iit} - I_{it}^{(IM)}, 0)/B_{-iit}$ denotes the international-investor access weight to issuer i's domestic market, and $a_{-iit}^{(IM)} = 1 - a_{-iit}^{(DM)}$ denotes the international-investor access weight to issuer i's international market.

Figure A2 illustrates access weights averaged over 2001–2021 for the top 20 issuer countries, separately for domestic and international investors. The upper panel shows that domestic investors generally buy domestic debt in domestic markets, although in Great Britain, the Netherlands, and Ireland they buy significant amounts of domestic debt issued in international markets. The lower panel shows that international investors generally buy international debt in international markets, although for the United States, China, Japan, and Belgium, they purchase significant amounts of debt issued in domestic markets. In 5.87% of country-year observations, weights fall outside the interval $[0, 1]$ and are truncated to zero or one.

Step Three: Issuance Weights. We compute issuance weights for 142 issuer countries that report international-market issuance in total and by currency for USD, EUR, and the issuer's domestic currency in the BIS IDS data. When values are present for at least one of these currencies, we set any remaining missing values to zero. We then define issuance in "other" currencies (neither USD, EUR, nor the issuer's domestic currency) as the remainder after subtracting these three currency totals from the overall total international-market issuance.[7]

Issuance weights for issuer-country i satisfy the following identity,

$$\begin{aligned} I_{it}^{(IM)} &= I_{iit}^{(IM)} + I_{iUSDt}^{(IM)} + I_{iEURt}^{(IM)} + I_{iOTHt}^{(IM)} \\ \Leftrightarrow \quad 1 &= v_{iit}^{(IM)} + v_{iUSDt}^{(IM)} + v_{iEURt}^{(IM)} + v_{iOTHt}^{(IM)}\,, \end{aligned} \tag{A7}$$

where $I_{it}^{(IM)}$ denotes total international-market issuance, $I_{ijt}^{(IM)}$ denotes issuance in currency j. We define the issuance weight for issuer country i and issuance currency j as $v_{ijt}^{(IM)} \equiv I_{ijt}^{(IM)}/I_{it}^{(IM)}$. In particular, $I_{iit}^{(IM)}$ and $v_{iit}^{(IM)}$ denote the issuance amount and weight for issuer country i's international-market issuance in its domestic currency. For domestic-market issuance, we assume that all debt is issued in the issuer's domestic currency, so that $v_{iit}^{(DM)} \equiv 1$.[8]

Step Four: Portfolio Weights. We compute portfolio weights for investor-country debt holdings by combining holding weights with issuer weights and access weights. Portfolio weights allow us to decompose each investor country's total holdings into holdings by issuer country and issuance currency. These weights play an important role in the calculation of portfolio returns on investor country debt holdings in section A.3.

We compute separate portfolio weights for domestic and international debt holdings.

[7] A large share of country-year observations (72.15%) have missing values for USD, EUR, or domestic currency issuance in the IDS data. However, after setting these missing values to zero, the average share of international market issuance in the other-currency category remains low at 6.23%, compared with 8.64% for observations with no missing values. This suggests that the zero-fill assumption does not artificially inflate the other-currency category.

[8] This assumption appears reasonable for most countries. Gruić and Wooldridge (2012) observe that "bonds issued in the local market are typically issued in local currency under local law," but note international financial centers and dollarised or euroised economies as possible exceptions.

For domestic debt holdings, portfolio weights are given by

$$w_{iikt} \equiv \begin{cases} b_{iit} a_{iit}^{(IM)} v_{ikt}^{(IM)} \,, & i \neq k \,, \\ b_{iit} \Big(a_{iit}^{(IM)} v_{iit}^{(IM)} + a_{iit}^{(DM)} \Big) \,, & i = k \,, \end{cases} \tag{A8}$$

where w_{iikt} denotes the portfolio weight for investor country i's holdings of domestic debt issued in currency k. For international debt holdings, portfolio weights are given by

$$w_{ijkt} \equiv \begin{cases} b_{ijt} a_{-jjt}^{(IM)} v_{jkt}^{(IM)} \,, & i \neq j \neq k \,, \\ b_{ijt} \Big(a_{-jjt}^{(IM)} v_{jjt}^{(IM)} + a_{-jjt}^{(DM)} \Big) \,, & i \neq j = k \,, \end{cases} \tag{A9}$$

where w_{ijkt} denotes the portfolio weight for investor country i's holdings of debt issued by country j in currency k, where $i \neq j$. For country-year observations where we lack IDS data and cannot compute access or issuance weights, we use the global average of non-missing access and issuance weights to compute portfolio weights.[9]

Step Five: Biproportional Fitting. Using the portfolio weights from step four, we can derive total international debt holdings by currency for each investor country. For a subset of 66 countries holding 80.20% of global debt in an average year, we can compare this derived total with a CPIS-reported total, and thereby obtain an indirect measure of accuracy for the underlying portfolio weights. To derive total international debt holdings by currency, we simply multiply total international debt holdings by portfolio weights and aggregate over issuers.

The upper panel of Figure A3 shows the discrepancy between our derived totals and the CPIS-reported totals by currency. With accurate portfolio weights, observations would fall on the 45-degree line. Instead, our portfolio weights overestimate international debt holdings in EUR and USD when the true values are low and underestimate when the true values are high. The opposite is true of CHF, GBP, and JPY, but these currencies represent a much smaller fraction of international debt.

To address this issue, we adjust initial portfolio weights using a biproportional fitting algorithm. We introduce the following notation. Let B_{ijkt} denote country i's holding of debt issued by country j in currency k, computed as a portfolio weight multiplied by country i's total holdings, and let $B_{ij\bullet t}$ and $B_{i\bullet kt}$ denote total debt holdings by issuer and

[9] Many of the smallest issuer countries lack IDS data. While these countries are numerous (causing 42.29% of portfolio weights to be computed with global averages), the debt they issue accounts for a negligible 0.29% of global holdings in an average year.

by currency, respectively. That is, let

$$B_{ijkt} = w_{ijkt} B_{it}\,, \quad B_{ij\bullet t} = \sum_k B_{ijkt}\,, \quad \text{and} \quad B_{i\bullet kt} = \sum_j B_{ijkt}\,. \tag{A10}$$

Note that $B_{ij\bullet t}$ has the same meaning as B_{ijt} used elsewhere; we adopt the "dot" notation here to avoid confusion with $B_{i\bullet kt}$. Finally, let $B^{\dagger}_{i\bullet kt}$ denote CPIS-reported (as opposed to derived) total holdings by currency.

For each investor–year combination, we arrange initial estimates of holdings by issuer and currency into a matrix, with issuers j in rows and currencies k in columns. We iteratively adjust rows and columns to obtain a set of holdings $B^{\dagger}_{ijkt}$ such that

$$B_{ij\bullet t} = \sum_k B^{\dagger}_{ijkt} \quad \text{and} \quad B^{\dagger}_{i\bullet kt} = \sum_j B^{\dagger}_{ijkt} \quad \forall j, k\,. \tag{A11}$$

The row target $B_{ij\bullet t}$ ensures that the issuer-level totals remain unchanged from our estimates in Section A.2.1, while the column target $B^{\dagger}_{i\bullet kt}$ forces the currency composition to align with the CPIS-reported values.

At iteration n, we first scale each row so that its sum equals the corresponding row target $B_{ij\bullet t}$. Specifically, for issuer (row) j, we update

$$B^{(n+\frac{1}{2})}_{ijkt} = B^{(n)}_{ijkt} \frac{B_{ij\bullet t}}{\sum_k B^{(n)}_{ijkt}}, \quad \forall\, k\,. \tag{A12}$$

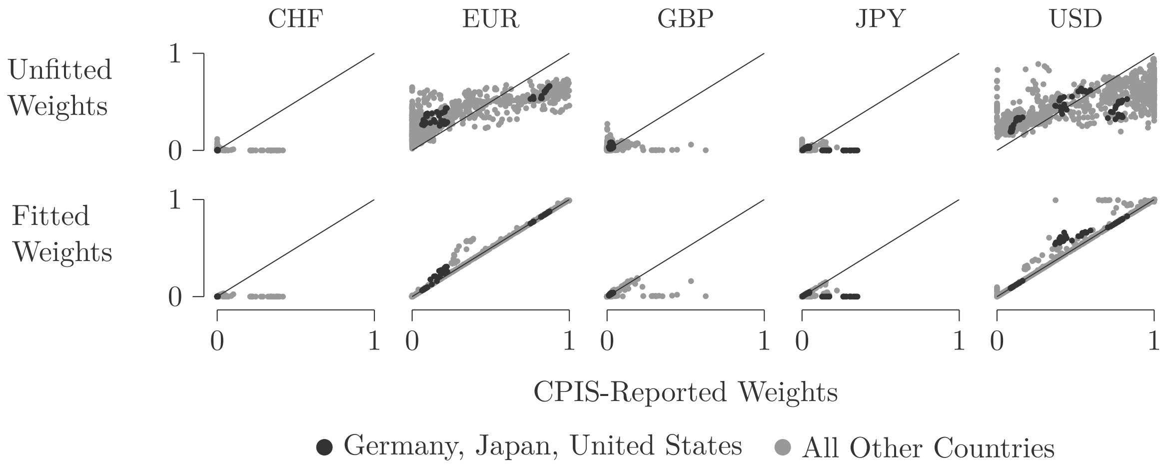
Next, we scale each column so that its sum equals the corresponding column target $B^{\dagger}_{i\bullet kt}$. Specifically, for currency (column) k, we update

$$B^{(n+1)}_{ijkt} = B^{(n+\frac{1}{2})}_{ijkt} \frac{B^{\dagger}_{i\bullet kt}}{\sum_j B^{(n+\frac{1}{2})}_{ijkt}}, \quad \forall\, j\,. \tag{A13}$$

These steps are repeated until the maximum deviation between the current row and column sums and their respective targets falls below a given tolerance. If convergence is not achieved within a maximum number of iterations, we apply a final re-scaling to enforce the row constraints. The final values $B^{\dagger}_{ijkt}$ are then used to update portfolio weights w_{ijkt}.

Convergence often fails because our initial estimates B_{ijkt} are derived from IDS data covering USD, EUR, and domestic currency, while our currency targets are derived from CPIS data covering USD, EUR, JPY, GBP, and CHF. In the IDS data, we construct a residual "other" category that includes JPY, GBP, and CHF when these are non-domestic currencies. As a result, the JPY, GBP, and CHF columns are sparsely populated, which prevents convergence to the CPIS-reported targets in these currencies. However, these currencies account for only 11.22% of total international debt holdings, and the algorithm is effective for USD and EUR. The lower panel of Figure A3 illustrates this effectiveness,

Figure A3 – Biproportional Fitting of Portfolio Weights to Currency Weights



Notes. The figure shows scatter plots of estimated and CPIS-reported currency weights for CHF, EUR, GBP, JPY, and USD from 2001 to 2021 before and after the biproportional fitting described in Section A.2.2, step five. Germany, Japan, and the United States are shown in black, and all other countries are shown in gray. The top row shows unfitted estimates, and the bottom row shows fitted estimates. Each plot includes a 45-degree reference line. After fitting, estimates for USD and EUR align closely with reported values. Estimates for GBP, CHF, and JPY remain less well aligned, but these currencies account for only 11.22% of international debt holdings.

with fitted values clustering around the 45-degree line for USD and EUR.

A.3 Domestic Currency Bias and Portfolio Returns

Table 1 reports average estimates of domestic currency bias and portfolio returns on debt holdings for Germany, Japan, and the United States as one group, and for 181 rest-of-world countries as another group. On average, Germany, Japan, and the United States exhibit lower portfolio returns and a stronger domestic-currency bias than other countries. In this appendix section, we describe the data and methodology used to estimate domestic currency bias and portfolio returns for country-level debt holdings.

Domestic Currency Bias. We compute domestic currency bias using the debt holdings and portfolio weights estimated in Section A.2. Following Fidora et al. (2007), we define

$$\begin{aligned} DCB_{it} &= 100\left(w^*_{-it} - w_{-it}\right)/w^*_{-it}\,, \quad \text{with} \\ w_{-it} &= \sum_j \sum_{k\neq i} w_{ijkt} \quad \text{and} \quad w^*_{-it} = \sum_j \sum_{k\neq i} w_{ijkt} B_{it} \Big/ \sum_i B_{it}\,, \end{aligned} \tag{A14}$$

where DCB_{it} denotes domestic currency bias for country i, w_{-it} denotes the share of international-currency debt in i's debt holdings, and w^*_{-it} denotes the share of international-currency debt in global debt holdings (with "international" defined relative to country i). Domestic currency bias is bounded above by 100 and unbounded below. It is negative in only 2.38% of country-year observations, indicating that most countries overweight domestic currency in their debt holdings.

Portfolio Returns. We compute nominal portfolio returns as a weighted average of issuer-country nominal interest rates adjusted for currency returns, taken over all positions in an investor country's debt holdings. Real portfolio returns equal nominal portfolio returns adjusted for the investor country's domestic rate of CPI inflation. We obtain interest rates from the Organization for Economic Cooperation and Development (OECD) Main Economic Indicators (MEI) database and IMF International Financial Statistics (IFS) database. Exchange rates and rates of CPI inflation are also from IFS.

Short-term interest rates serve as indicative interest rates for our portfolio return calculations.[10] For each issuer country, we use OECD MEI short-term interest rates where available, and IMF IFS treasury bill rates where MEI rates are unavailable.[11] These two sources cover 2 069 country-year observations. For an additional 1 032 country-year observations, these sources are unavailable but IFS deposit rates are available. In these cases, we use a fitted model to predict short-term interest rates from deposit rates,

$$R^i_{Bit} = \alpha + \beta_D R^i_{Dit} + \boldsymbol{\beta}'_{FE}\boldsymbol{FE}_{it} + \epsilon_t \,, \tag{A15}$$

where R^i_{Bit} and R^i_{Dit} denote gross short-term and deposit rates, respectively, with subscript i denoting the issuer country and superscript i denoting the numéraire currency, set as the domestic currency of the issuing country, and $\boldsymbol{FE}_{it}$ denotes country and year fixed effects. The regression yields an $R^2 = 0.82$ and coefficient estimates $\hat{\beta}_D = 0.01$ and $\hat{\alpha} = 98.83$, both significant at the 1% level. In total, reported or predicted rates are available for 3101 country-year observations.

End-of-year nominal exchange rates from IFS are available for 3 778 country-year observations, reported with respect to USD. With the nominal exchange rate between currency i and USD denoted $S_{i\text{USD}t}$, we define the gross nominal currency return for an arbitrary currency pair as

$$R_{Sijt} = R_{Si\text{USD}t} / R_{Sj\text{USD}t} \,, \quad \text{where} \quad R_{Si\text{USD}t} = S_{i\text{USD}t} / S_{i\text{USD}t-1} \quad \forall\, i \,. \tag{A16}$$

[10] Our estimates cover debt of all maturities. Ideally, we would match interest rate terms to debt maturities when computing returns, but short-term and long-term debt data are less complete in BIS and IMF sources. For robustness, Panel A3c of Table A3 reports portfolio returns for short-term debt using available data.

[11] MEI short-term rates are "based on three-month money market rates where available. Typical standardised names are 'money market rate' and 'treasury bill rate' " (OECD, 2025). IMF treasury bill rates are rates "at which short-term government debt securities are issued or traded in the market," with details varying slightly from country to country (International Monetary Fund, 2023a,b).

Table A2 – Summary of Domestic Currency Bias and Portfolio Returns

Variable		Summary Statistics (in %)			
	Count	Mean	SD	Median	IQR
Portfolio Return, Nominal	3 778	5.6	8.8	3.9	6.5
Portfolio Return, Real	3 459	0.5	7.1	0.4	5.0
Coverage Ratio	3 778	84.7	25.6	95.6	8.8
Domestic Currency Bias	3 778	62.9	29.0	73.1	42.0
Currency Return	3 778	2.3	11.8	0.0	7.8
CPI Inflation Rate	3 459	5.3	9.4	3.3	5.1

Notes. The table presents summary statistics for country-year estimates of domestic currency bias, portfolio returns, and coverage ratios in the upper panel, and for the variables used in the calculation of portfolio returns: treasury bill rates, currency returns, and CPI rates of inflation in the lower panel. Statistics are computed for the period 2001–2021. The count column counts non-missing country-year observations. Mean, standard deviation (SD), median, and interquartile range (IQR) are presented in percentage.

We define the nominal portfolio return R^i_{NPit} for country i in numéraire currency i as

$$R^i_{NPit} = \sum_j \sum_k w_{ijkt} R_{Sikt} R^j_{Bjt}\,, \tag{A17}$$

where we apply the domestic short-term interest rate R^j_{Bjt} to debt issued by j in any currency.[12] We then use domestic rates of CPI inflation for the investor country to convert nominal portfolio returns into real portfolio returns, $R_{Pit} = R^i_{NPit}/\Pi^i_{it}$.

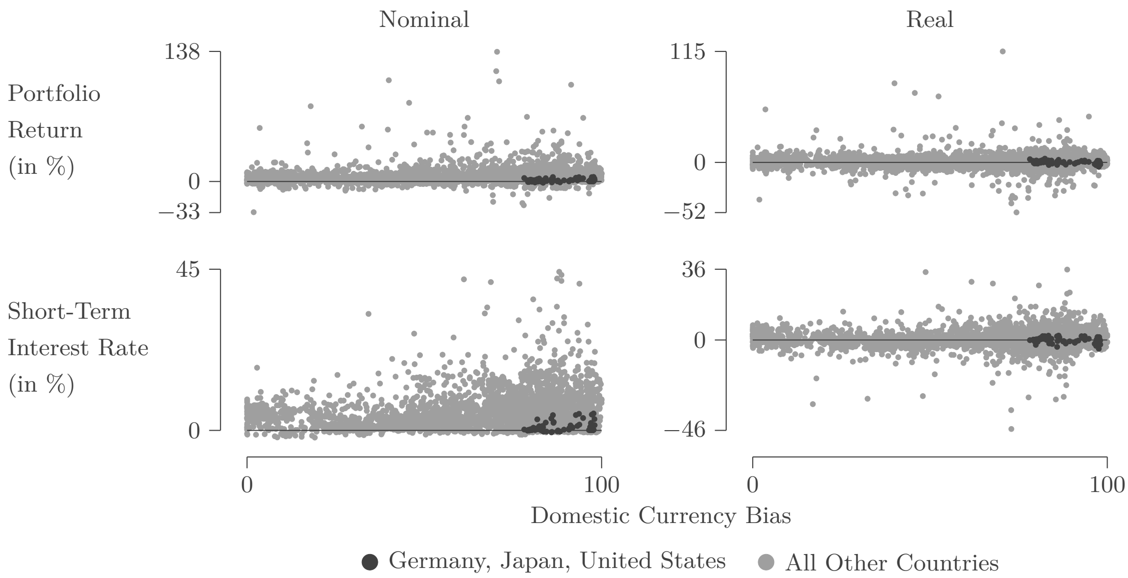
For some debt-issuing countries and years, short-term interest rates or currency returns are unavailable. For this reason, we compute portfolio returns for each investor country on the fraction of portfolio holdings where data are available, re-scaling portfolio weights by the coverage ratio, which we define as

$$CR_{it} = \sum_j \sum_k w_{ijkt} \mathbb{1}_{ijk} \tag{A18}$$

where the indicator $\mathbb{1}_{ijk}$ equals zero when interest rates or exchange rates are missing for investor country i, issuer country j, or currency k. The median coverage ratio in our sample is 95.6%. Table A2 provides descriptive statistics for portfolio returns, coverage ratios, domestic currency bias, currency returns, and rates of inflation in our sample.

[12] The IMF and OECD have poor coverage of interest rates on debt issued in non-domestic currency. Empirically, interest rates on debt issued in domestic and international currencies do differ; for reference, the ECB reports a median international-minus-domestic currency rate spread of +0.8pp (advanced economies) and −1.6pp (emerging economies) (European Central Bank, 2014b, Section C). While these spreads are large, they would be multiplied by small portfolio weights in our analysis, and would not significantly impact the main empirical findings we report in Table 1. Du and Schreger (2016, 2022) study the spread for sovereign debt in emerging markets, emphasizing currency risk, credit risk, and corporate balance sheet mismatches as important determinants, which our simplification does not capture.

Figure A4 – Domestic Currency Bias and Portfolio Returns



Notes. The figure shows domestic currency bias and both nominal and real short-term debt portfolio returns and treasury rates for 188 countries, estimated annually. Germany, Japan, and the United States are shown in black; all other countries are shown in gray. Portfolio returns are calculated using short-term interest rates, exchange rates, and CPI inflation rates from the OECD and IMF, with portfolio weights estimated from IMF and BIS debt holdings and issuance data. Domestic currency bias is defined in equation (A14). We exclude 2.38% of estimates with negative domestic currency bias and country-year observations with interest or exchange rates more than three standard deviations above the yearly mean.

Figure A4 presents scatter plots of portfolio returns and short-term interest rates against domestic currency bias. The horizontal axes show domestic currency bias. In the upper plots, the vertical axis shows portfolio returns, nominal on the left and real on the right. In the lower plots, the vertical axis shows domestic short-term interest rates, nominal on the left and real on the right. Germany, Japan, and the United States are shown in black, and their observations cluster near zero on the vertical axis and near 100 on the horizontal axis. All other countries are shown in gray and exhibit more variation in domestic currency bias, portfolio returns, and domestic short-term interest rates.

Table A3 provides three robustness checks for the result in Table 1 that Germany, Japan, and the United States have higher domestic currency bias and lower portfolio returns than other countries (91.37% vs. 72.96% bias; −0.27% vs. 0.73% real returns). Panel A3a shows the pattern holds with unweighted averages (88.60% vs. 62.44% bias; −0.05% vs. 0.55% real returns). Panel A3b shows the pattern holds when including outliers (91.37% vs. 72.93% bias; −0.26% vs. 0.80% real returns). Panel A3c shows the pattern holds for short-term debt only (99.52% vs. 95.29% bias; −0.68% vs. 0.14% real returns). In all specifications, the qualitative result of higher domestic currency bias and lower returns for Germany, Japan, and the United States remains unchanged.

Table A3 – Domestic Currency Bias and Returns on Aggregate Debt Portfolios

(a) Weighted versus Unweighted Country Averages, Outliers Excluded, All Debt

	Weighted Average	Group Share of Global Holdings	Country-Level Average (in %) Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return	Portfolio Coverage Ratio
DE, JP, US	Yes	53.97	91.37	1.20	−0.27	97.89
All Other	Yes	46.03	72.96	3.16	0.73	94.02
DE, JP, US	No	53.97	88.60	1.21	−0.05	97.70
All Other	No	46.03	62.44	5.71	0.55	84.47

(b) Weighted Country Averages, Outliers Excluded versus Included, All Debt

	Outliers Excluded	Group Share of Global Holdings	Country-Level Weighted Average (in %) Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return	Portfolio Coverage Ratio
DE, JP, US	Yes	53.97	91.37	1.20	−0.27	97.89
All Other	Yes	46.03	72.96	3.16	0.73	94.02
DE, JP, US	No	53.94	91.37	1.21	−0.26	97.91
All Other	No	46.06	72.93	3.34	0.80	94.15

(c) Weighted Country Averages, Outliers Excluded, All Debt versus Short-Term Debt Only

	Short Term Only	Group Share of Global Holdings	Country-Level Weighted Average (in %) Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return	Portfolio Coverage Ratio
DE, JP, US	No	53.97	91.37	1.20	−0.27	97.89
All Other	No	46.03	72.96	3.16	0.73	94.02
DE, JP, US	Yes	52.27	99.52	1.20	−0.68	99.92
All Other	Yes	47.73	95.29	3.15	0.14	92.06

Notes. This table presents three robustness checks for Table 1, which reports domestic currency bias and returns on debt portfolios for Germany, Japan, and the United States versus 181 other countries. Panel A3a compares weighted versus unweighted country averages. Panel A3b compares results with and without outliers (interest rates and exchange rate returns exceeding three standard deviations from annual means). Panel A3c compares all debt maturities versus short-term debt only. All estimates use OECD and IMF data for interest rates, exchange rates, and CPI inflation, with portfolio weights derived from IMF and BIS debt holdings and issuance data (2001–2021). Global holdings shares are averaged across years by group; other values are averaged across countries and years, applying debt holdings as weights where indicated.

B Theoretical Appendix

Appendix B provides derivations and proofs for the model in Section 4. It covers parameter restrictions (B.1), real and nominal budget constraint (B.2), utility maximization (B.3), the non-stochastic steady state (B.4), first-order Taylor expansions of equilibrium conditions (B.5), model aggregability (B.6), the aggregate non-portfolio problem (B.7) and portfolio problem (B.8), and the agent non-portfolio problem (B.9) and portfolio problem (B.10).

B.1 Parameter Restrictions

In this section, we collect our assumptions on parameter restrictions for easier reference throughout the appendix.

Assumption 1 (Preference Parameters)**.** *The preference parameters unrelated to risk aversion satisfy*

$$\theta \in [0, \varepsilon)\,, \quad \beta \in (0.5, 1)\,, \quad \textit{and} \quad \alpha_{ii} \in (0.5, 1)\,, \tag{B1}$$

where $\alpha_{ii} + \alpha_{ij} = 1$ *so that domestic bias in consumption* $\phi = \alpha_{ii} - \alpha_{ij}$ *lies in* $(0, 1)$.

These restrictions ensure that individual preferences yield a well-defined steady state and support the emergence of domestic portfolio bias in the model. Agents favor consumption of the domestic good when $\alpha_{ii} \in (0.5, 1)$, which strengthens the incentive to hold domestic bonds and helps to produce domestic bias in aggregate portfolio holdings.

Assumption 2 (Distribution of Risk Aversion)**.** *We assume risk aversion* ρ *lies in the interval* $(0, \rho_m]$, *and that the parameters of the cross-sectional distribution of risk aversion satisfy*

$$\kappa > 1 \quad \textit{and} \quad \rho_m > (\kappa^2 + \kappa)/(\kappa^2 - 1)\,, \tag{B2}$$

ensuring that the distribution has finite mean and variance, and that aggregate risk aversion is sufficiently high ($\bar{\rho} > \omega$)*.*

Sufficiently high risk aversion is needed for Propositions 8, 9, and 10 establishing domestic portfolio bias in aggregate, negative expected cross-county differenced returns, and the existence of carry traders in low interest-rate economies.

Assumption 3 (Endowments and Money Supplies)**.** *The logarithmic deviations of endowments and money supplies from their non-stochastic steady-state values have zero means, positive and finite variances and covariances, and follow independent, identically distributed processes over time. With* $\hat{\boldsymbol{Z}}_t \equiv [\hat{Y}_{it},\, \hat{Y}_{jt},\, \hat{M}^i_{it},\, \hat{M}^j_{jt}]'$, $i \neq j$, *we assume*

$$\mathrm{E}_{t-1}\big[\hat{\boldsymbol{Z}}_t \hat{\boldsymbol{Z}}_t'\big] = \begin{bmatrix} \sigma_Y^2 & \sigma_{YY} & \sigma_{YM} & \sigma_{YM}^* \\ \sigma_{YY} & \sigma_Y^2 & \sigma_{YM}^* & \sigma_{YM} \\ \sigma_{YM} & \sigma_{YM}^* & \sigma_M^2 & \sigma_{MM} \\ \sigma_{YM}^* & \sigma_{YM} & \sigma_{MM} & \sigma_M^2 \end{bmatrix}, \tag{B3}$$

where $\sigma_{YM} > \sigma^*_{YM}$, $\sigma^2_Y > \sigma_{YY}$, and $\sigma^2_M > \sigma_{MM}$, ensuring that domestic covariances exceed cross-country covariances and that neither endowment shocks nor money supplies are perfectly correlated across countries.

Assumption 3, which is stated verbally in section 4.1, preserves cross-country symmetry, and helps to produce low expected returns on portfolios with domestic bias. However, our results also hold under less restrictive assumptions. Specifically, all propositions, including Propositions 8, 9, and 10, hold if log endowments and money supplies are imperfectly correlated across countries and

$$\mathrm{E}_t\Big[\hat{Y}_{i-jt+1}\hat{Y}_{i+jt+1}\Big] \geq 0\,, \quad \mathrm{E}_t\Big[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i-jt+1}\Big] \geq 0\,,$$
$$\text{and} \quad \mathrm{E}_t\Big[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i+jt+1}\Big] \geq 0\,, \quad i \neq j\,.$$

Though not our focus, the less-restrictive conditions do admit asymmetric variance-covariance structures that can generate persistent differences in returns across countries.

B.2 Nominal and Real Budget Constraints

In this section, we relate the nominal agent budget constraint to the real agent budget constraint in (6), and the nominal bond return to the real bond return. Nominal and real budget constraints are mathematically equivalent and lead to identical first-order conditions.

Substituting the real return and real portfolio holdings into the agent's real budget constraint in (6) using the definitions in (8), we obtain

$$P^i_{it}\frac{A_{iit}(\rho)P^i_{Bit}}{P^i_{it}} + P^i_{it}\frac{A_{ijt}(\rho)P^i_{Bjt}}{P^i_{it}} + P^i_{it}C_{it}(\rho)$$
$$= P^i_{it}\frac{S_{iit}/P^i_{it}}{P^i_{Bit-1}/P^i_{it-1}}\frac{A_{iit-1}(\rho)P^i_{Bit-1}}{P^i_{it-1}} + P^i_{it}\frac{S_{ijt}/P^i_{it}}{P^i_{Bjt-1}/P^i_{it-1}}\frac{A_{ijt-1}(\rho)P^i_{Bjt-1}}{P^i_{it-1}} + P^i_{Cit}Y_{it}(\rho)\,, \quad i \neq j\,,$$

which simplifies to the agent's nominal budget constraint,

$$A_{iit}(\rho)P^i_{Bit} + A_{ijt}(\rho)P^i_{Bjt} + P^i_{it}C_{it}(\rho) = A_{iit-1}(\rho) + S_{ijt}A_{ijt-1}(\rho) + P^i_{Cit}Y_{it}(\rho)\,, \quad i \neq j\,.$$

The agent's Euler equation in (28) would remain unchanged if we were to derive it from the nominal instead of the real budget constraint, maximizing with respect bond quantities rather than real values of bond holdings.

The Euler equations in (28) can be written in terms of nominal instead of real returns using the following expression,

$$
\begin{aligned}
R^j_{it} &= \frac{S_{jit}/P^j_{jt}}{P^j_{Bit-1}/P^j_{jt-1}} \\
&= \frac{S_{jit}/P^j_{Bit-1}}{P^j_{jt}/P^j_{jt-1}} = R^j_{Bit}/\Pi^j_{it}\,,
\end{aligned}
\tag{B4}
$$

where R^j_{Bit} denotes the gross nominal return and Π^j_{it} denotes the gross rate of inflation. A version of the Fisher equation obtains as a first-order Taylor expansion of (B4),

$$
\hat{R}^j_{Bit} = \hat{R}^j_{it} + \hat{\Pi}^j_t + O(\epsilon^2)\,. \tag{B5}
$$

B.3 Utility Maximization

For the first-stage utility maximization problem, agent ρ in country i solves

$$
\begin{aligned}
\max\ \mathcal{L}^{(1)}_{it}(\rho) = &\sum_{s=t}^{\infty} \beta^{s-t}\,\mathrm{E}_t\left[\frac{1}{1-\rho}\left(\frac{C_{is}(\rho)}{C_i(\rho)}\right)^{1-\rho} + \frac{\theta}{1-\rho}\left(\frac{e^{W^i_{is}(\rho)}}{e^{W^i_i(\rho)}}\right)^{1-\rho}\right] \\
&+ \sum_{s=t}^{\infty} \beta^{s-t}\,\mathrm{E}_t\Big[\mu^{(1)}_{is}(\rho)\Big\{R^i_{is}B^i_{iis-1}(\rho) + R^i_{js}B^i_{ijs-1}(\rho) + W^i_{is}(\rho)\Big\}\Big] \\
&- \sum_{s=t}^{\infty} \beta^{s-t}\,\mathrm{E}_t\Big[\mu^{(1)}_{is}(\rho)\Big\{B^i_{iis}(\rho) + B^i_{ijs}(\rho) + C_{is}(\rho)\Big\}\Big]\,, \quad i \neq j
\end{aligned}
\tag{B6}
$$

by choice of $\left\{C_s(\rho), B^i_{iis}(\rho), B^i_{ijs}(\rho)\right\}$ for $i \neq j$ and for $s \geq t$, where $\mathcal{L}^{(1)}_{it}(\rho)$ denotes the agent's Lagrangian for the first-stage problem and $\mu^{(1)}_{it}(\rho)$ denotes the Lagrangian multiplier on the agent's budget constraint. First-order conditions with respect to consumption and real bond holdings are given by

$$
\begin{aligned}
\mu^{(1)}_{it}(\rho) &= \left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho}\frac{1}{C_i(\rho)} \quad \text{and} \\
\mu^{(1)}_{it}(\rho) &= \theta\left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho} + \beta\,\mathrm{E}_t\Big[\mu^{(1)}_{it+1}(\rho)R^i_{jt+1}\Big]\,.
\end{aligned}
\tag{B7}
$$

Combining the first-order conditions in (B7) yields the agent Euler equations in (16).

For the second-stage utility maximization problem, agent ρ in country i solves

$$
\begin{aligned}
\max\ \mathcal{L}^{(2)}_{it}(\rho) = &\,\gamma C_{iit}(\rho)^{\alpha_{ii}} C_{ijt}(\rho)^{\alpha_{ij}} \\
&+ \mu^{(2)}_{it}(\rho)\Big(P^j_{it}C_{it}(\rho) - P^i_{Cit}C_{iit}(\rho) - P^i_{Cjt}C_{ijt}(\rho)\Big)\,, \quad i \neq j
\end{aligned}
\tag{B8}
$$

by choice of $C_{iit}(\rho)$ and $C_{ijt}(\rho)$ for $i \neq j$, where $\mathcal{L}^{(2)}_{it}(\rho)$ denotes the agent's Lagrangian for the second-stage problem and $\mu^{(2)}_{it}(\rho)$ denotes the Lagrangian multiplier on the agent's expenditure constraint in (11). First-order conditions with respect to the domestic and international consumption goods are given by

$$\mu_{it}^{(2)}(\rho)P_{Cit}^{i} = \gamma\alpha_{ii}\left(\frac{C_{ijt}(\rho)}{C_{iit}(\rho)}\right)^{\alpha_{ij}} \quad \text{and} \quad \mu_{it}^{(2)}(\rho)P_{Cjt}^{i} = \gamma\alpha_{ij}\left(\frac{C_{iit}(\rho)}{C_{ijt}(\rho)}\right)^{\alpha_{ii}}, \quad i \neq j\,. \tag{B9}$$

Combining (B9) with the expenditure constraint in (11) yields the goods demands in (17). Combining (B9) with the definition of the consumption basket in (10) yields the price index in (18).

B.4 The Non-Stochastic Steady State

In this section, we prove Propositions 1 and 2 characterizing the non-stochastic steady-state equilibrium of the model.

B.4.1 Proof of Proposition 1

Proposition 1 characterizes the non-portfolio non-stochastic steady state. We begin by deriving the steady-state price indices, goods prices, and the real exchange rate. From the quantity equations and the equality of steady-state money supplies in (15),

$$M_i^i = P_{Ci}^i Y_i = S_{ij} P_{Cjt}^j Y_j = S_{ij} M_j^j = M_j^i\,. \tag{B10}$$

Using the normalization of steady-state real endowments in (9) together with the law of one price in (18), it follows that common-currency prices for the domestic and international goods are equal,

$$P_{Ci}^i = P_{Cj}^i\,, \tag{B11}$$

and using the price indices in (18) and the common-currency equality of goods prices, it follows that price indices and goods prices are equal,

$$P_i^j = P_{Ci}^j\,. \tag{B12}$$

Using the equality of prices derived above together with the definition of the real exchange rate in (14) evaluated at the steady state,

$$Q_{ij} = \frac{S_{ij}P_j^j}{P_i^i} = \frac{P_j^i}{P_i^i} = \frac{P_{Cj}^i}{P_{Ci}^i} = \frac{P_{Cj}^i}{P_{Cj}^i} = 1\,. \tag{B13}$$

We now derive steady-state consumption. The left-hand side of the steady-state Euler equation in (19) is independent of j, so real returns on domestic and international nominal bonds are equal,

$$R_i^i = R_i^j\,, \tag{B14}$$

and the right-hand side is independent of ρ, so agent and aggregate consumption baskets are equal (given that the measure of agents equals one in each country),

$$C_i(\rho) = C_i \,. \tag{B15}$$

From the goods demands in (17), together with steady-state prices in (21),

$$C_{ij}(\rho) = C_{ij} = \alpha_{ij} C_i \,, \tag{B16}$$

and together with goods market clearing in (12), the equality of agent and aggregate endowments in (9), and the parameter restrictions in (1),

$$C_i(\rho) = C_i = Y_i = Y_i(\rho) = 1 \,. \tag{B17}$$

Since $C_i(\rho) = 1$, it follows from (19) that

$$R_i^j = \frac{1-\theta}{\beta} = R \,. \tag{B18}$$

We now derive steady-state real wealth. From the budget constraints in (6), using the definition of real wealth in (7) and the equality of steady-state real returns in (22),

$$W_i^i(\rho) + C_i(\rho) = RW_i^i(\rho) + Y_i(\rho) \,. \tag{B19}$$

Using the normalized values of steady-state consumption and endowments, and rearranging, the budget constraint becomes

$$W_i^i(\rho)(1-R) = 0 \,. \tag{B20}$$

Given that the measure of agents equals one in each country, it follows from the budget constraint that

$$W_i^i(\rho) = W_i^i = W_j^j = W_j^j(\rho) \,. \tag{B21}$$

From the bond market clearing conditions in (12), summing across bonds,

$$B_{ii}^i + B_{ji}^i + B_{jj}^i + B_{ij}^i = 0 \quad \Leftrightarrow \quad W_i^i + W_j^i = 0 \,, \quad i \neq j \,, \tag{B22}$$

using the definition of real wealth in (7). From the definitions of real wealth in (7) and real bond holdings in (8), together with the steady-state real exchange rate in (21),

$$W_j^i = Q_{ij} W_j^j = W_j^j \,, \tag{B23}$$

Hence, we have

$$W_i^i = W_j^j\,, \quad W_i^i + W_j^i = 0\,, \quad \text{and} \quad W_j^i = W_j^j\,, \tag{B24}$$

and therefore

$$W_i^j(\rho) = W_i^j = 0\,. \tag{B25}$$

B.4.2 Proof of Proposition 2

Proposition 2 characterizes the portfolio non-stochastic steady state. The characterization is limited, and derives from the solution for steady-state agent and aggregate real wealth and from market clearing. From steady-state real wealth in (22), using the definition of real wealth in (7), we have

$$B_{ii}^i(\rho) = -B_{ij}^i(\rho) \quad \text{and} \quad B_{ii}^i = -B_{ij}^i\,. \tag{B26}$$

Combining these results with market clearing in (12), we have

$$B_{ii}^i = B_{jj}^i\,. \tag{B27}$$

We derive more complete characterizations of steady-state agent and aggregate portfolio holdings (to order-zero approximations) in Proposition 10.

B.5 First-Order Taylor Expansions

Several Taylor expansions already appear in the body of the paper; these include agent Euler equations in (28), agent budget constraints in (30), agent goods demands in (31), and market clearing conditions for goods and bonds in (32). For the remaining equilibrium conditions, Taylor expansions are given below:

$$\begin{aligned}
&\text{Agent Real Wealth:} && \hat{W}_{it}^j(\rho) = \hat{B}_{iit}^j(\rho) + \hat{B}_{ijt}^j(\rho) + O(\epsilon^2)\,, \quad i \neq j\,,\\
&\text{Real Bond Returns:} && \hat{R}_{it}^j = \left(\hat{S}_{jit} - \hat{P}_{Bit-1}^j\right) - \left(\hat{P}_{it}^j - \hat{P}_{it-1}^j\right) + O(\epsilon^2)\,,\\
&\text{Price Indices:} && \hat{P}_{it}^k = \alpha_{ii}\hat{P}_{Cit}^k + \alpha_{ij}\hat{P}_{Cjt}^k + O(\epsilon^2)\,, \quad i \neq j\,,\\
&\text{Consumption LoP:} && \hat{P}_{Cit}^i = \hat{S}_{ijt} + \hat{P}_{Cit}^j + O(\epsilon^2)\,,\\
&\text{Bond LoP:} && \hat{P}_{Bit}^i = \hat{S}_{ijt} + \hat{P}_{Bit}^j + O(\epsilon^2)\,,\\
&\text{Real Exchange Rate:} && \hat{Q}_{ijt} = \hat{S}_{ijt} + \hat{P}_{jt}^j - \hat{P}_{it}^i + O(\epsilon^2)\,, \quad \text{and}\\
&\text{Quantity Equations:} && \hat{M}_{it}^j = \hat{Y}_{it} + \hat{P}_{it}^j + O(\epsilon^2)\,.
\end{aligned} \tag{B28}$$

B.6 Model Aggregability

In this section, we derive expressions for the aggregate coefficient of relative risk aversion and the aggregate intertemporal elasticity of substitution, and prove Propositions 3 and 4.

B.6.1 Aggregating Preference Parameters

Given the CRRA consumption utility in (4), heterogeneity can be defined equivalently in terms of either the coefficient of relative risk aversion or the intertemporal elasticity of substitution with respect to consumption. Using the distribution in (2), the aggregate coefficient of relative risk aversion and intertemporal elasticity of substitution are given by

$$\bar{\rho} = \int_{\mathcal{R}} \rho f(\rho)\,\mathrm{d}\rho = \frac{\kappa \rho_m}{\kappa + 1} \quad \text{and} \quad \bar{\sigma} = \int_{\mathcal{R}} \sigma(\rho) f(\rho)\,\mathrm{d}\rho = \frac{\kappa \rho_m}{\kappa - 1}\,, \tag{B29}$$

where we have used equation (5) for $\sigma(\rho)$.

B.6.2 Proof of Proposition 3

Proposition 3 introduces the aggregation wedge ω. Using the expressions for $\bar{\rho}$ and $\bar{\sigma}$ in (B29) and the definition of ω in (27), we have

$$\bar{\sigma} = \frac{\omega}{\bar{\rho}} \quad \Leftrightarrow \quad \frac{\kappa \rho_m}{\kappa - 1} = \omega \frac{\kappa + 1}{\kappa \rho_m} \quad \Leftrightarrow \quad \omega = \frac{\kappa^2}{\kappa^2 - 1}\,.$$

By inspection, the wedge is greater than one and decreasing in κ for $\kappa > 1$.

B.6.3 Proof of Proposition 4

Proposition 4 states the first-order approximate agent and aggregate Euler equations. We begin with the agent Euler equation. Combining fist-order conditions in (B7), we obtain

$$\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho} \frac{1}{C_i(\rho)} = \theta \left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho} + \beta\,\mathrm{E}_t\left[\left(\frac{C_{it+1}(\rho)}{C_i(\rho)}\right)^{-\rho} \frac{1}{C_i(\rho)} R^i_{jt+1}\right].$$

A standard first-order Taylor expansion of this expression around the non-stochastic steady state then yields (28).

The expression in (28) aggregates straightforwardly. The coefficient of relative risk aversion is the only source heterogeneity in the expression, and it multiplies an aggregate variable. Integrating (28) over agents,

$$\begin{aligned} \int_{\mathcal{R}} \hat{C}_{it}(\rho) f(\rho)\,\mathrm{d}\rho &= \theta \int_{\mathcal{R}} \hat{W}^i_{it}(\rho) f(\rho)\,\mathrm{d}\rho \\ &+ \beta R\,\mathrm{E}_t\left[\int_{\mathcal{R}} \hat{C}_{it+1}(\rho) f(\rho)\,\mathrm{d}\rho - \int_{\mathcal{R}} \frac{1}{\rho} f(\rho)\,\mathrm{d}\rho \hat{R}^i_{jt+1}\right] + O(\epsilon^2)\,, \end{aligned} \tag{B30}$$

and noting that (25) implies

$$\hat{C}_{it} = \int_{\mathcal{R}} \hat{C}_{it}(\rho) f(\rho)\,\mathrm{d}\rho + O(\epsilon^2) \quad \text{and} \quad \hat{W}^j_{it} = \int_{\mathcal{R}} \hat{W}^j_{it}(\rho) f(\rho)\,\mathrm{d}\rho + O(\epsilon^2)\,,$$

we obtain (29) from (B30) directly. Expressions in (5) and (26) are used for the coefficient of relative risk aversion.

B.7 The Aggregate Non-Portfolio Problem

In the aggregate non-portfolio problem we solve for aggregate consumption and real wealth and for the real exchange rate in terms of lagged state variables, exogenous variables, and parameters. We first derive cross-country differenced solutions, and then use market clearing conditions to derive country-specific solutions.

B.7.1 The Aggregate Non-Portfolio System

To derive the cross-country differenced aggregate non-portfolio system in (36), we combine the market clearing condition in (32) with the aggregate budget constraints in (30) differenced across countries and aggregate Euler equations in (29) differenced across countries. Using the market clearing condition in (32) to eliminate the real exchange rate from the cross-country differenced aggregate budget constraint in (34), we obtain

$$\hat{W}^i_{i-jt} = R\hat{W}^i_{i-jt-1} - \frac{1-\phi}{1-\phi^2}\hat{C}_{i-jt} + \phi\frac{1-\phi}{1-\phi^2}\hat{Y}_{i-jt} + R\hat{V}^i_{i-jt} + O(\epsilon^2)\,. \tag{B31}$$

Similarly, using the market clearing condition in (32) to eliminate the real exchange rate from the cross-country differenced aggregate Euler equation in (34), we obtain

$$\begin{aligned}\theta\hat{W}^i_{i-jt} + \beta R\left(1+\bar{\sigma}\frac{\phi^2}{1-\phi^2}\right)\mathrm{E}_t\left[\hat{C}_{i-jt+1}\right] \\ = \left(1+\bar{\sigma}\beta R\frac{\phi^2}{1-\phi^2}\right)\hat{C}_{i-jt} + \bar{\sigma}\beta R\frac{\phi}{1-\phi^2}\hat{Y}_{i-jt} + O(\epsilon^2)\,.\end{aligned} \tag{B32}$$

These two equations, (B31) and (B32), constitute the cross-country differenced aggregate system. The system depends on three endogenous variables: cross-country differenced aggregate consumption $\hat{C}_{i-jt}$, cross-country differenced aggregate real wealth $\hat{W}^i_{i-jt}$, and the cross-country differenced aggregate portfolio valuation effect $\hat{V}^i_{i-jt}$. We solve the system initially for $\hat{C}_{i-jt}$ and $\hat{W}^i_{i-jt}$ in terms of $\hat{V}^i_{i-jt}$, describing these solutions as *intermediate solutions*. Solving for $\hat{V}^i_{i-jt}$ requires second-order Taylor expansions of Euler equations and a solution for B^i_{ii-ji}, which we derive in B.8.

We write the cross-country differenced aggregate system in (B31) and (B32) as

$$\boldsymbol{\mathcal{E}}^{(-)}_0\,\mathrm{E}_t\left[\hat{\boldsymbol{Z}}_{i-jt+1}\right] = \boldsymbol{\mathcal{E}}^{(-)}_1\hat{\boldsymbol{Z}}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_2\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_3\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{B33}$$

where the coefficient matrices are given by

$$\boldsymbol{\mathcal{E}}_0^{(-)} = \begin{bmatrix} 1 & 0 \\ \theta & \beta R\left(1+\bar{\sigma}\frac{\phi^2}{1-\phi^2}\right) \end{bmatrix}, \quad \boldsymbol{\mathcal{E}}_1^{(-)} = \begin{bmatrix} R & -\frac{1-\phi}{1-\phi^2} \\ 0 & 1+\bar{\sigma}\beta R\frac{\phi^2}{1-\phi^2} \end{bmatrix},$$
$$\boldsymbol{\mathcal{E}}_2^{(-)} = \begin{bmatrix} \phi\frac{1-\phi}{1-\phi^2} \\ -\bar{\sigma}\beta R\frac{\phi}{1-\phi^2} \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\mathcal{E}}_3^{(-)} = \begin{bmatrix} R \\ 0 \end{bmatrix}. \tag{B34}$$

The matrix $\boldsymbol{\mathcal{E}}_0^{(-)}$ is invertible if its determinant is non-zero. Therefore, we require that

$$\det\left(\boldsymbol{\mathcal{E}}_0^{(-)}\right) = \beta R\left(1+\bar{\sigma}\frac{\phi^2}{1-\phi^2}\right) \neq 0\,, \tag{B35}$$

which is always satisfied under Assumptions 1 and 2. Inverting $\boldsymbol{\mathcal{E}}_0^{(-)}$, we rewrite the aggregate system in (B33) to obtain (36), with partial elasticity matrices given by

$$\boldsymbol{\mathcal{E}}_{ZZ}^{(-)} = \begin{bmatrix} \mathcal{E}_{WW}^{(-)} & \mathcal{E}_{WC}^{(-)} \\ \mathcal{E}_{CW}^{(-)} & \mathcal{E}_{CC}^{(-)} \end{bmatrix} = \begin{bmatrix} R & -\frac{1-\phi}{1-\phi^2} \\ -\frac{\theta}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2} & \frac{1}{\beta R}\frac{1+\theta(1-\phi)-(1-\bar{\sigma}\beta R)\phi^2}{1-(1-\bar{\sigma})\phi^2} \end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}_{ZY}^{(-)} = \begin{bmatrix} \mathcal{E}_{WY}^{(-)} \\ \mathcal{E}_{CY}^{(-)} \end{bmatrix} = \begin{bmatrix} \phi\frac{1-\phi}{1-\phi^2} \\ -\frac{\phi}{1-(1-\bar{\sigma})\phi^2}\left(\bar{\sigma}+\frac{\theta(1-\phi)}{\beta R}\right) \end{bmatrix}, \quad \text{and} \tag{B36}$$

$$\boldsymbol{\mathcal{E}}_{ZV}^{(-)} = \begin{bmatrix} \mathcal{E}_{WV}^{(-)} \\ \mathcal{E}_{CV}^{(-)} \end{bmatrix} = \begin{bmatrix} R \\ -\frac{\theta}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2} \end{bmatrix}.$$

Before solving the aggregate cross-country differenced system in (36), we establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}_{ZZ}^{(-)}$ is given by

$$\boldsymbol{\mathcal{E}}_{ZZ}^{(-)} = \boldsymbol{\mathcal{V}}_{ZZ}^{(-)}\boldsymbol{\Lambda}_{ZZ}^{(-)}\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}} \tag{B37}$$

where $\boldsymbol{\mathcal{V}}_{ZZ}^{(-)}$ is a matrix of eigenvectors and $\boldsymbol{\Lambda}_{ZZ}^{(-)}$ a diagonal matrix of eigenvalues,

$$\boldsymbol{\mathcal{V}}_{ZZ}^{(-)} = \begin{bmatrix} \nu_{WW}^{(-)} & \nu_{WC}^{(-)} \\ \nu_{CW}^{(-)} & \nu_{CC}^{(-)} \end{bmatrix} = \begin{bmatrix} \mathcal{E}_{WC}^{(-)} & \mathcal{E}_{WC}^{(-)} \\ \lambda_{ZW}^{(-)} - \mathcal{E}_{WW}^{(-)} & \lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)} \end{bmatrix}, \quad \text{and}$$
$$\boldsymbol{\Lambda}_{ZZ}^{(-)} = \begin{bmatrix} \lambda_{ZW}^{(-)} & 0 \\ 0 & \lambda_{ZC}^{(-)} \end{bmatrix}, \tag{B38}$$

where

$$\lambda_{ZW}^{(-)} = \frac{1}{2}\left(\mathcal{E}_{WW}^{(-)} + \mathcal{E}_{CC}^{(-)}\right) + \sqrt{\frac{1}{4}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}}\,, \quad \text{and} \tag{B39}$$

$$\lambda_{ZC}^{(-)} = \frac{1}{2}\left(\mathcal{E}_{WW}^{(-)} + \mathcal{E}_{CC}^{(-)}\right) - \sqrt{\frac{1}{4}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}}\,. \tag{B40}$$

B.7.2 Proof of Proposition 5

Proposition 5 states that $\lambda_{ZW}^{(-)}$ and $\lambda_{ZC}^{(-)}$ are distinct and real-valued if $\theta \geq 0$. To prove this statement, it suffices to show that the discriminant in (B39) and (B40) is positive. The discriminant Δ is given by

$$\Delta = \frac{1}{4}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}\,. \tag{B41}$$

Suppose $\theta > 0$. The squared term is always positive or zero, so we need to show that the second term is positive,

$$\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)} > 0 \quad \Leftrightarrow \quad \frac{\theta}{\beta}\frac{1-\phi}{1-(1-\bar{\sigma})\phi^2} > 0\,, \tag{B42}$$

where we have used the partial elasticities in (B36). Under Assumptions 1 and 2, the inequality in (B42) is always satisfied.

Suppose $\theta = 0$. The second term is zero, $\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)} = 0$, but the squared term is positive, since $\mathcal{E}_{WW}^{(-)}\big|_{\theta=0} - \mathcal{E}_{CC}^{(-)}\big|_{\theta=0} = 1/\beta - 1 > 0$, so the discriminant Δ is again positive. Thus, $\theta \geq 0$ is sufficient for distinct and real-valued eigenvalues.

The case where $\theta = 0$ serves as a helpful benchmark for proving the remaining statements in Proposition 5. Table B1 reports the partial elasticities in (36) and their derivatives with respect to θ, evaluated at $\theta = 0$.

Proposition 5 states that $\lambda_{ZW}^{(-)}$ evaluated at $\theta = 0$ is strictly greater than one and $\lambda_{ZC}^{(-)}$ evaluated at $\theta = 0$ is exactly one. Using Table B1 to evaluate (B39) at $\theta = 0$,

$$\lambda_{ZW}^{(-)}\Big|_{\theta=0} = \frac{1}{2}\left(\frac{1}{\beta} + 1\right) + \frac{1}{2}\left|\frac{1}{\beta} - 1\right| = \frac{1}{\beta} = \mathcal{E}_{WW}^{(-)}\Big|_{\theta=0} > 1\,, \tag{B43}$$

and (B40) at $\theta = 0$,

$$\lambda_{ZC}^{(-)}\Big|_{\theta=0} = \frac{1}{2}\left(\frac{1}{\beta} + 1\right) - \frac{1}{2}\left|\frac{1}{\beta} - 1\right| = \mathcal{E}_{CC}^{(-)}\Big|_{\theta=0} = 1\,. \tag{B44}$$

Proposition 5 also states that the partial derivative of $\lambda_{ZW}^{(-)}$ with respect to θ evaluated at $\theta = 0$ is positive, and the partial derivative of $\lambda_{ZC}^{(-)}$ with respect to θ evaluated at $\theta = 0$

Table B1 – Aggregate Partial Elasticities and Their Derivatives at $\theta = 0$

Partial Elasticity	Value of Partial Elasticity Evaluated at $\theta=0$		Derivative with Respect to θ Evaluated at $\theta=0$	
	Expression	Sign	Expression	Sign
$\mathcal{E}^{(-)}_{WW}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{WC}$	$-\frac{1-\phi}{1-\phi^2}$	$-$	0	$+$
$\mathcal{E}^{(-)}_{CW}$	0	$\cdot$	$-\frac{1}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2}$	$-$
$\mathcal{E}^{(-)}_{CC}$	1	$+$	$\frac{2-\phi-\phi^2}{1-(1-\bar{\sigma})\phi^2}$	$+$
$\mathcal{E}^{(-)}_{WY}$	$\phi\frac{1-\phi}{1-\phi^2}$	$+$	0	$\cdot$
$\mathcal{E}^{(-)}_{CY}$	$-\frac{\phi\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}$	$-$	$-\frac{\phi(1-\phi)}{1-(1-\bar{\sigma})\phi^2}$	$-$
$\mathcal{E}^{(-)}_{WV}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{CV}$	0	$\cdot$	$-\frac{1}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2}$	$-$

Notes. Column one lists the partial elasticities that appear in the cross-country differenced aggregate system (36). Columns two and four, respectively, give expressions for the partial elasticities and their derivatives with respect to θ, both evaluated at $\theta = 0$. Columns three and five, respectively, give the signs of the partial elasticities and their derivatives, both evaluated at $\theta = 0$. Signs are established under Assumptions 1 and 2.

is negative. To establish this result, we differentiate the eigenvalues with respect to θ,

$$\begin{aligned}\frac{\partial\lambda^{(-)}_{\pm}}{\partial\theta} &= \frac{1}{2}\left(\frac{\partial\mathcal{E}^{(-)}_{ZW}}{\partial\theta} + \frac{\partial\mathcal{E}^{(-)}_{ZC}}{\partial\theta}\right) \pm \frac{1}{2}\left[\frac{1}{4}\left(\mathcal{E}^{(-)}_{WW} - \mathcal{E}^{(-)}_{CC}\right)^2 + \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CW}\right]^{-\frac{1}{2}} \\ &\times \left[\frac{1}{2}\left(\mathcal{E}^{(-)}_{WW} - \mathcal{E}^{(-)}_{CC}\right)\left(\frac{\partial\mathcal{E}^{(-)}_{WW}}{\partial\theta} - \frac{\partial\mathcal{E}^{(-)}_{CC}}{\partial\theta}\right) + \frac{\partial\mathcal{E}^{(-)}_{WC}}{\partial\theta}\mathcal{E}^{(-)}_{CW} + \mathcal{E}^{(-)}_{WC}\frac{\partial\mathcal{E}^{(-)}_{CW}}{\partial\theta}\right],\end{aligned} \tag{B45}$$

where the "+" case corresponds to eigenvalue $\lambda^{(-)}_{ZW}$ and the "−" case corresponds to eigenvalue $\lambda^{(-)}_{ZC}$. Using Table B1, we then evaluate (B45) at $\theta = 0$,

$$\begin{aligned}\left.\frac{\partial\lambda^{(-)}_{ZW}}{\partial\theta}\right|_{\theta=0} &= \left.\frac{\partial\mathcal{E}^{(-)}_{WW}}{\partial\theta}\right|_{\theta=0} + \frac{\beta}{1-\beta}\left.\mathcal{E}^{(-)}_{WC}\right|_{\theta=0}\left.\frac{\partial\mathcal{E}^{(-)}_{CW}}{\partial\theta}\right|_{\theta=0} \\ &= \frac{1}{1-\beta}\frac{1-\phi}{1-(1-\bar{\sigma})\phi^2} - \frac{1}{\beta},\end{aligned} \tag{B46}$$

and

$$\begin{aligned}\left.\frac{\partial\lambda^{(-)}_{ZC}}{\partial\theta}\right|_{\theta=0} &= \left.\frac{\partial\mathcal{E}^{(-)}_{CC}}{\partial\theta}\right|_{\theta=0} - \frac{\beta}{1-\beta}\left.\mathcal{E}^{(-)}_{WC}\right|_{\theta=0}\left.\frac{\partial\mathcal{E}^{(-)}_{CW}}{\partial\theta}\right|_{\theta=0} \\ &= \frac{1}{1-(1-\bar{\sigma})\phi^2}\left[2-\phi-\phi^2-\frac{1-\phi}{1-\beta}\right].\end{aligned} \tag{B47}$$

The right-hand side of (B46) is positive if

$$\beta > \frac{1-(1-\bar{\sigma})\phi^2}{2-(1-\bar{\sigma})\phi^2-\phi}\,, \tag{B48}$$

and the right-hand side of (B47) is negative if

$$\beta > \frac{1-\phi^2}{2-\phi^2-\phi}\,. \tag{B49}$$

The inequality in (B49) is implied by the inequality in (B48). Hence, (B48) provides a sufficient condition for the derivatives of $\lambda_{ZW}^{(-)}$ and $\lambda_{ZC}^{(-)}$ with respect to θ to be positive and negative, respectively, when evaluated at $\theta = 0$.

Assumptions 1 and 2 ensure that (B48) is satisfied. By continuity, a small positive value for θ then pushes the eigenvalues $\lambda_{ZC}^{(-)}$ below one and $\lambda_{ZW}^{(-)}$ further above one, ensuring a unique and stationary rational expectations solution to (36).

B.7.3 Intermediate Aggregate Non-Portfolio Solution

We now derive intermediate solutions for aggregate real wealth, aggregate consumption, and the real exchange rate in terms lagged state variables, exogenous variables and parameters, and the endogenous aggregate portfolio valuation effect. Using the eigendecomposition in (B37), we rewrite the aggregate cross-country differenced system in (B33) as

$$\begin{aligned}&\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}\Big]\\ &\quad=\boldsymbol{\Lambda}_{ZZ}^{(-)}\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\hat{\boldsymbol{Z}}_{i-jt}+\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\boldsymbol{\mathcal{E}}_{ZY}^{(-)}\hat{Y}_{i-jt}+\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\boldsymbol{\mathcal{E}}_{ZV}^{(-)}\hat{V}_{i-jt}^{i}+O(\epsilon^2)\,,\end{aligned} \tag{B50}$$

and from (B50) we extract the equation associated with the unstable eigenvalue $\lambda_{ZW}^{(-)}$,

$$\begin{aligned}\mathrm{E}_t\Big[\hat{Z}_{i-jt+1}\Big]=\lambda_{ZW}^{(-)}\hat{Z}_{i-jt}&+\Big[\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WY}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}\Big]\hat{V}_{i-jt}^{i}\\ &+\Big[\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WV}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}\Big]\hat{Y}_{i-jt}+O(\epsilon^2)\,,\end{aligned} \tag{B51}$$

where $\hat{Z}_{i-jt+1}=\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\hat{W}_{i-jt}^{i}-\mathcal{E}_{WC}^{(-)}\hat{C}_{i-jt+1}$.

Because $\lambda_{ZW}^{(-)}$ is greater than one, the left-hand side of (B51) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\hat{C}_{i-jt}=\eta_{CW}^{(-)}\hat{W}_{i-jt-1}^{i}+\eta_{CY}^{(-)}\hat{Y}_{i-jt}+\eta_{CV}^{(-)}\hat{V}_{i-jt}^{i}+O(\epsilon^2)\,, \tag{B52}$$

and using (B52) to eliminate $\hat{C}_{i-jt}$ from (B31), we obtain

$$\hat{W}_{i-jt}^{i}=\eta_{WW}^{(-)}\hat{W}_{i-jt-1}^{i}+\eta_{WY}^{(-)}\hat{Y}_{i-jt}+\eta_{WV}^{(-)}\hat{V}_{i-jt}^{i}+O(\epsilon^2)\,. \tag{B53}$$

Writing the intermediate solutions in (B52) and (B53) in more compact matrix form, we obtain (38), where the semi-partial elasticity matrices are given by

$$\boldsymbol{\eta}_{ZW}^{(-)} = \begin{bmatrix} \eta_{WW}^{(-)} \\ \eta_{CW}^{(-)} \end{bmatrix} = \begin{bmatrix} \lambda_{ZC}^{(-)} \\ \frac{\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}}{\mathcal{E}_{WC}^{(-)}} \end{bmatrix},$$

$$\boldsymbol{\eta}_{ZY}^{(-)} = \begin{bmatrix} \eta_{WY}^{(-)} \\ \eta_{CY}^{(-)} \end{bmatrix} = \begin{bmatrix} \frac{\left(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\right)\mathcal{E}_{WY}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}}{\lambda_{ZW}^{(-)}} - \phi\mathcal{E}_{WC}^{(-)} \\ \frac{\left(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\right)\mathcal{E}_{WY}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}} \end{bmatrix}, \quad \text{and} \tag{B54}$$

$$\boldsymbol{\eta}_{ZV}^{(-)} = \begin{bmatrix} \eta_{WV}^{(-)} \\ \eta_{CV}^{(-)} \end{bmatrix} = \begin{bmatrix} \frac{\left(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\right)\mathcal{E}_{WV}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}}{\lambda_{ZW}^{(-)}} + \mathcal{E}_{WW}^{(-)} \\ \frac{\left(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\right)\mathcal{E}_{WV}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}} \end{bmatrix}.$$

For the real exchange rate, we combine the market clearing condition in (32) with our intermediate solution for cross-country differenced consumption in (B52) to obtain (42), with semi-partial elasticities

$$\begin{aligned} \eta_{QW}^{(-)} &= -\frac{\phi^2}{1-\phi^2}\frac{\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}}{\mathcal{E}_{WC}^{(-)}} \\ \eta_{QY}^{(-)} &= \frac{\phi}{1-\phi^2}\left(1 - \phi\frac{\left(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\right)\mathcal{E}_{WY}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}}\right), \quad \text{and} \\ \eta_{QV}^{(-)} &= -\frac{\phi^2}{1-\phi^2}\frac{\left(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\right)\mathcal{E}_{WV}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}}. \end{aligned} \tag{B55}$$

The semi-partial elasticities in (B54) and (B55) can be written in terms of model parameters using the partial elasticities in (B36) and the eigenvalues in (B39) and (B40).

B.8 The Aggregate Portfolio Problem

In the aggregate portfolio problem we solve for cross-country differenced realized and expected real returns on nominal bonds and cross-country differenced aggregate portfolio holdings of nominal bonds.

To begin, we take the cross-country difference of realized real returns in (B28) to obtain

$$\hat{R}_{i-jt+1}^{i} = -\hat{S}_{jit+1} - \hat{P}_{Bi-jt}^{i} + O(\epsilon^2), \tag{B56}$$

which we combine with the first-order expansion of expected real returns in (47) to obtain

$$\hat{R}^i_{i-jt+1} = -\Big(S_{ijt+1} - \mathrm{E}_t\big[S_{ijt+1}\big]\Big) + O(\epsilon^2)\,. \tag{B57}$$

Using first-order Taylor expansions of the real exchange rate, price indices, and quantity equations in 47, the nominal exchange rate can be written as

$$\hat{S}_{ijt} = \frac{1}{\phi}\hat{Q}_{ijt} + \hat{M}^{i-j}_{i-jt} - \hat{Y}_{i-jt} + O(\epsilon^2)\,, \tag{B58}$$

and combining with the intermediate solution for the real exchange rate in (42) and the expression for the cross-country differenced real return in (B57), we obtain

$$\hat{R}^i_{i-jt} = \eta^{(-)}_{RY}\hat{Y}_{i-jt} + \eta^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + \eta^{(-)}_{RV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{B59}$$

where the real return semi-partial elasticities are given by

$$\begin{aligned}
\eta^{(-)}_{RY} &= \frac{\phi}{1-\phi^2}\left(\frac{\Big(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\Big)\mathcal{E}^{(-)}_{WY} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CY}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}} - \phi\right), \\
\eta^{(-)}_{RM} &= -1\,, \text{ and} \\
\eta^{(-)}_{RV} &= \frac{\phi}{1-\phi^2}\frac{\Big(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\Big)\mathcal{E}^{(-)}_{WV} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CV}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}}\,.
\end{aligned} \tag{B60}$$

We substitute the portfolio valuation effect (35) to into (B59) and rearrange to obtain the cross-country differenced real return in terms of general elasticities in (43), where the general elasticities are given by

$$\gamma^{(-)}_{RY} = \frac{\eta^{(-)}_{RY}}{1-\eta^{(-)}_{RV}B^i_{ii-ji}} \quad \text{and} \quad \gamma^{(-)}_{RM} = \frac{\eta^{(-)}_{RM}}{1-\eta^{(-)}_{RV}B^i_{ii-ji}}\,, \tag{B61}$$

and where the semi-partial elasticities $\eta^{(-)}_{RY}$, $\eta^{(-)}_{RM}$, and $\eta^{(-)}_{RV}$ can be written in terms of model parameters using the partial elasticities in (B36), but where B^i_{ii-ji} remains to be solved. For use later, we report the semi-partial elasticities from the aggregate portfolio problem in Table B2, evaluate at $\theta = 0$.

B.8.1 Proof of Proposition 6

The cross-country differenced aggregate portfolio valuation multiplier $\mu^{(-)}_{ii-ji}$ in (44) depends on the semi-partial elasticity $\eta^{(-)}_{RV}$ in (B60), and exceeds one in absolute value if

$$\frac{1}{\Big|1-\eta^{(-)}_{RV}B^i_{ii-ji}\Big|} > 1\,. \tag{B62}$$

Table B2 – Aggregate Semi-Partial Elasticities at $\theta = 0$

Semi-Partial Elasticity	Evaluation at $\theta = 0$	
	Expression	Sign
$\eta^{(-)}_{CY}$	$(1-\beta)\phi + \frac{\beta\phi\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}$	$+$
$\eta^{(-)}_{WY}$	$\beta\phi\frac{(1-\phi)(1-\bar{\sigma})}{1-(1-\bar{\sigma})\phi^2}$	$+$
$\eta^{(-)}_{QY}$	$\phi\left(1+\beta\frac{1-\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}\right)$	$+$
$\eta^{(-)}_{RY}$	$-\beta\phi^2\frac{1-\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}$	$-$
$\eta^{(-)}_{CV}$	$\frac{1-\beta}{\beta}\frac{1-\phi^2}{1-\phi}$	$+$
$\eta^{(-)}_{WV}$	$\frac{1-\phi^2-(1-\beta)(1-\phi)}{\beta(1-\phi^2)}$	$+$
$\eta^{(-)}_{QV}$	$-\frac{1-\beta}{\beta}\frac{\phi^2}{1-\phi}$	$-$
$\eta^{(-)}_{QW}$	$-\frac{1-\beta}{\beta}\frac{\phi^2}{1-\phi}$	$-$
$\eta^{(-)}_{RV}$	$\frac{1-\beta}{\beta}\frac{1-\phi^2}{1-\phi}$	$+$
$\eta^{(-)}_{RM}$	-1	$-$

Notes. The table shows semi-partial elasticities from the cross-country differenced aggregate non-portfolio and portfolio problems, evaluated at $\theta = 0$. Column two give an expression for each elasticity, and column three gives its sign under Assumptions 1 and 2.

From Table B2, $\eta^{(-)}_{RV} > 0$ at $\theta = 0$. Because $\eta^{(-)}_{RV}$ is continuous in θ in the neighborhood of $\theta = 0$, its sign is preserved for $\theta \in [0, \epsilon)$.

Case one: $1 - \eta^{(-)}_{RV} B^i_{ii-ji} > 0$. In this case, $1 > 1 - \eta^{(-)}_{RV} B^i_{ii-ji}$ is equivalent to $0 < B^i_{ii-ji}$. Hence, $0 < B^i_{ii-ji} < 1/\eta^{(-)}_{RV}$ implies $\left|\mu^{(-)}_{ii-ji}\right| > 1$. Case two: $1 - \eta^{(-)}_{RV} B^i_{ii-ji} < 0$. In this case, $1 > \eta^{(-)}_{RV} B^i_{ii-ji} - 1$ is equivalent to $B^i_{ii-ji} < 2/\eta^{(-)}_{RV}$. Hence, $\frac{1}{\eta^{(-)}_{RV}} < B^i_{ii-ji} < \frac{2}{\eta^{(-)}_{RV}}$ implies $\left|\mu^{(-)}_{ii-ji}\right| > 1$. Combining cases one and two,

$$0 < B^i_{ii-ji} < \frac{2}{\eta^{(-)}_{RV}} \quad \Rightarrow \quad \left|\mu^{(-)}_{ii-ji}\right| > 1 \,. \tag{B63}$$

As an illustration, suppose $\theta = 0$, $\phi = 0.5$, and $\beta = 0.95$. In this case, $2/\eta^{(-)}_{RV} = 38$; domestic bias in aggregate portfolio holdings amplifies the response of the cross-country differenced real return to endowment and money supply shocks, provided that the cross-country differenced aggregate holdings of country i's bond are less than 38 times the size of the steady-state economy, which is normalized to one.

In contrast, $\mu^{(-)}_{ii-ji}$ lies between zero and one if

$$0 \leq \frac{1}{1 - \eta^{(-)}_{RV} B^i_{ii-ji}} < 1 \,. \tag{B64}$$

Given that $\eta^{(-)}_{RV}$ is positive, these inequalities are satisfied for any $B^i_{ii-ji} < 0$. In other words, any amount of international bias in aggregate portfolio holdings attenuate rather than amplify the response of the cross-country differenced real return to endowment and money supply shocks.

B.8.2 Proof of Proposition 7

Second-order Taylor expansions of the agent Euler equations in (16) are given by

$$\begin{aligned}&\hat{C}_{it}(\rho) - \frac{\rho}{2}\hat{C}_{it}(\rho)^2 - \theta\hat{W}^i_{it}(\rho) + \frac{\theta\rho}{2}\hat{W}^i_{it}(\rho)\\ &\quad = \beta R\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho) - \frac{\rho}{2}\hat{C}_{it+1}(\rho)^2 - \frac{1}{\rho}\hat{R}^i_{jt+1} + \hat{C}_{it+1}(\rho)\hat{R}^i_{jt+1}\Big] + O(\epsilon^3)\,.\end{aligned} \tag{B65}$$

Differencing (B65) across bonds, i minus j, for agent ρ in country i, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1}\Big] + O(\epsilon^3)\,. \tag{B66}$$

We use this second-order approximate agent Euler equation to obtain the cross-country differenced and cross-country summed Euler equations that appear in Proposition 7.

We begin by deriving the second-order approximate cross-country differenced agent Euler equations in (48). Differencing (B66) across countries for agents with identical coefficients of relative risk aversion, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} - \hat{R}^j_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1} - \hat{C}_{jt+1}(\rho)\hat{R}^j_{i-jt+1}\Big] + O(\epsilon^3). \tag{B67}$$

The cross-country differenced real return in numéraire currency j, denoted $\hat{R}^j_{i-jt+1}$, now appears on both sides of the cross-country differenced Euler equations in (B67), which we convert to numéraire currency i before proceeding. To preserve the order of approximation in (B67), we need a second-order approximate conversion for the left-hand side of (B67) and a first-order approximate conversion for the right-hand side. Using the definition of the real return in (8) and the definition of the real exchange rate in (14), we obtain

$$\begin{aligned}\hat{R}^j_{i-jt+1} &= \hat{R}^i_{i-jt+1} + O(\epsilon^2) \quad \text{and}\\ \hat{R}^j_{i-jt+1} &= \hat{R}^i_{i-jt+1} - \hat{R}^i_{i-jt+1}\Big(\hat{Q}_{ijt+1} - \hat{Q}_{ijt}\Big) + O(\epsilon^3)\,.\end{aligned} \tag{B68}$$

Using (B68) to convert $\hat{R}^j_{i-jt+1}$ to numéraire currency i in (B67), we obtain the second-order approximate cross-country differenced agent Euler equation in equation (48).

We next derive the second-order approximate cross-country summed agent Euler equations in (48). Summing (B66) across countries for agents with identical coefficients of relative risk aversion ρ, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} + \hat{R}^j_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1} + \hat{C}_{jt+1}(\rho)\hat{R}^j_{i-jt+1}\Big] + O(\epsilon^3)\,. \tag{B69}$$

Again using (B68) to convert $\hat{R}^{j}_{i-jt+1}$ to numéraire currency i, we obtain the second-order approximate cross-country summed agent Euler equation in (48).

To obtain aggregate second-order approximate cross-country differenced and summed agent Euler equations, we exploit the fact that ρ multiplies no agent variables in (48) and integrate with respect to ρ, leading straightforwardly to the expressions in (49).

B.8.3 Proof of Proposition 8

To prove Proposition 8 on domestic bias in aggregate portfolio holdings, we first derive a final solution for cross-country differenced portfolio holdings, which we omitted from Section 4.2.5.

We derive the intermediate solution in (50) by combining the aggregate cross-country differenced portfolio valuation effect in (35), cross-country differenced aggregate consumption in (38) and the real exchange rate in (41) with the cross-country differenced aggregate Euler equations (49). This solution is intermediate because it depends on $\hat{R}^{i}_{i-jt+1}$.

Using the first-order approximate solution for $\hat{R}^{i}_{i-jt+1}$ in (42), and rearranging to isolate B^{i}_{ii-ji} on the left-hand side, the final solution is given by,

$$B^{i}_{ii-ji} = \frac{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_1}\left(\hat{Y}_{i-jt}\right)^2 + \zeta^{(-)}_{B_2}\left(\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right)\right]}{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_3}\left(\hat{Y}_{i-jt}\right)^2 + \zeta^{(-)}_{B_4}\left(\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right) + \zeta^{(-)}_{B_5}\left(\hat{M}^{i-j}_{i-jt}\right)^2\right]} + O(\epsilon)\,, \tag{B70}$$

where the coefficients given by

$$\begin{aligned}
\zeta^{(-)}_{B_1} &= \eta^{(-)}_{RY}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right), \\
\zeta^{(-)}_{B_2} &= \eta^{(-)}_{RM}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right), \\
\zeta^{(-)}_{B_3} &= \eta^{(-)}_{RY}\eta^{(-)}_{RV}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right) - \left(\eta^{(-)}_{RY}\right)^2\left(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{QV}\right), \\
\zeta^{(-)}_{B_4} &= \eta^{(-)}_{RM}\eta^{(-)}_{RV}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right) - 2\eta^{(-)}_{RM}\eta^{(-)}_{RY}\left(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{QV}\right), \text{ and} \\
\zeta^{(-)}_{B_5} &= -\left(\eta^{(-)}_{RM}\right)^2\left(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{RV}\right),
\end{aligned} \tag{B71}$$

and where the semi-partial elasticities that appear in the coefficients are defined in equations (B54), (B55), and (B60). The coefficients in (B71) are continuous in θ in the neighborhood of $\theta = 0$, so it will suffice for the proof to establish signs for the coefficients at $\theta = 0$, because these signs are preserved for θ sufficiently close to zero.

To establish signs for the coefficients in (B71), we must first establish signs for the semi-partial elasticities that appear in the coefficients. The semi-partial elasticities are defined in (B54), (B55), and (B60), and their values evaluated at $\theta = 0$ are given in Table B2. Using the table, we can further establish signs of two compound terms that appear repeatedly in the coefficients in (B71),

$$\left(\eta_{CY}^{(-)} - \bar{\sigma}\eta_{QY}^{(-)}\right)\Big|_{\theta=0} > 0 \quad \text{and} \quad \left(\eta_{CV}^{(-)} - \bar{\sigma}\eta_{RV}^{(-)}\right)\Big|_{\theta=0} > 0\,, \tag{B72}$$

under Assumptions 1 and 2.

Using these terms, Table B2, and (B72), we find that the coefficients in (B71) are negative when evaluated at $\theta = 0$:

$$\begin{gathered}\zeta_{B_1}^{(-)}\Big|_{\theta=0} < 0\,, \quad \zeta_{B_2}^{(-)}\Big|_{\theta=0} < 0\,, \quad \zeta_{B_3}^{(-)}\Big|_{\theta=0} < 0 \\ \zeta_{B_4}^{(-)}\Big|_{\theta=0} < 0\,, \quad \text{and} \quad \zeta_{B_5}^{(-)}\Big|_{\theta=0} < 0\,.\end{gathered} \tag{B73}$$

Assumption 3, together with the results in (B70) and (B73), are sufficient to conclude that $B_{ii-ji}^{i} > 0$. From the definition of cross-country differenced steady-state bond holdings in (35) and the non-stochastic steady-state portfolio equilibrium in Proposition 2, it follows that $B_{ij}^{i} < 0 + O(\epsilon) < B_{ii}^{i}$ as stated in Proposition 8.

B.8.4 Proof of Proposition 9

Proposition 9 states that the cross-country differenced real return is negative. To establish this result, we begin from the intermediate solution for expected cross-country differenced real returns in (54). We need to establish the signs for the three terms under expectation operators on the right-hand side of this expression, as well as for the coefficients on these terms.

We begin with the coefficients. The coefficient $\omega\bar{\rho}/2$ is positive by inspection, under Assumption 2. The signs of the remaining two coefficients are determined by the signs of the semi-partial elasticities $\eta_{QY}^{(-)}$ and $\eta_{QV}^{(-)}$. From Table B2, $\eta_{QY}^{(-)}$ and $\eta_{QV}^{(-)}$ are positive and negative at $\theta = 0$, respectively, under Assumptions 1 and 2. By continuity, these signs are preserved for small positive values of θ.

It remains to establish the signs of the three terms under expectation operators on the right-hand side of (54). The first term is $\mathrm{E}_t\left[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i+jt+1}\right]$. We use the cross-country differenced realized return in (43) to obtain

$$\mathrm{E}_t\left[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i+jt+1}\right] = \gamma_{RY}^{(-)}\,\mathrm{E}_t\left[\hat{Y}_{i-jt+1}\hat{Y}_{i+jt+1}\right] + \gamma_{RM}^{(-)}\,\mathrm{E}_t\left[\hat{M}_{i-jt+1}^{i-j}\hat{Y}_{i+jt+1}\right] + O(\epsilon^3)\,,$$

Under Assumption 3, this term is negative if the general elasticities $\gamma_{RY}^{(-)}$ and $\gamma_{RM}^{(-)}$ are negative. These general elasticities are given in (44). Their numerators $\eta_{RY}^{(-)}$ and $\eta_{RM}^{(-)}$ are negative at $\theta = 0$, as shown in Table B2. Their denominator $1 - \eta_{RV}^{(-)}B_{ii-ji}^{i}$ is positive if

$$\eta_{RV}^{(-)}B_{ii-ji}^{i} < 1\,,$$

or, using the solution for B^{i}_{ii-ji} in (B70), if

$$\begin{aligned}&\left(\eta^{(-)}_{CV}-\bar{\sigma}\eta^{(-)}_{QV}\right)\left(\eta^{(-)}_{RY}\right)^{2}\operatorname{E}_t\left[\left(\hat{Y}_{i-jt+1}\right)^{2}\right]\\&\quad+2\left(\eta^{(-)}_{CV}-\bar{\sigma}\eta^{(-)}_{QV}\right)\eta^{(-)}_{RY}\eta^{(-)}_{RM}\operatorname{E}_t\left[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i-jt+1}\right]>\zeta^{(-)}_{B_5}\operatorname{E}_t\left[\left(\hat{M}^{i-j}_{i-jt+1}\right)^{2}\right].\end{aligned} \tag{B74}$$

Evaluating this inequality at $\theta=0$ using the semi-partial elasticity signs from Table B2, and using the signs for the cross-country differenced bond holding coefficients in (B73), the left-hand side of (B74) is positive while the right-hand side is negative under Assumptions 1–3. By continuity, the inequality is preserved for values of θ in the neighborhood of zero. Thus, $\gamma^{(-)}_{RY}<0$ and $\gamma^{(-)}_{RM}<0$, and therefore $\operatorname{E}_t\left[\hat{R}^{i}_{i-jt+1}\hat{Y}_{i+jt+1}\right]\leq 0$.

The second term is $\operatorname{E}_t\left[\hat{R}^{i}_{i-jt+1}\hat{Y}_{i-jt+1}\right]$. We again use the cross-country differenced real returns in (43) to obtain

$$\operatorname{E}_t\left[\hat{R}^{i}_{i-jt+1}\hat{Y}_{i-jt+1}\right]=\gamma^{(-)}_{RY}\operatorname{E}_t\left[\left(\hat{Y}_{i-jt+1}\right)^{2}\right]+\gamma^{(-)}_{RM}\operatorname{E}_t\left[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i-jt+1}\right]+O(\epsilon^{3}),$$

and using $\gamma^{(-)}_{RY}<0$ and $\gamma^{(-)}_{RM}<0$ from above, we immediately obtain $\operatorname{E}_t\left[\hat{R}^{i}_{i-jt+1}\hat{Y}_{i-jt+1}\right]<0$ under Assumptions 1–3. The third term, $\operatorname{E}_t\left[\left(\hat{R}^{i}_{i-jt+1}\right)^{2}\right]$, is squared and therefore non-negative.

The signs of the three coefficients and three terms under expectation operators on the right-hand side of (54) under Assumptions 1–3 imply that $\operatorname{E}_t\left[\hat{R}^{i}_{i-jt+1}\right]<0+O(\epsilon^{3})$, as stated in Proposition 9. Importantly, these assumptions were also sufficient for domestic bias in aggregate portfolio holdings, as shown in Proposition 8. Propositions 8 and 9 together capture an empirical pattern that we observe in Germany, Japan, and the United States and document in Table 1 and Figure A4: strong domestic bias in aggregate portfolio holdings despite relatively low domestic returns.

B.9 The Agent Non-Portfolio Problem

In the agent non-portfolio problem we solve for agent consumption and real wealth in terms of exogenous variables and parameters. Unlike the aggregate system, where we solved for cross-country differences and used market clearing conditions to derive country-specific solutions, the agent system requires us to solve for both cross-country differences and cross-country sums. We derive these solutions in the following two subsections.

B.9.1 The Agent Non-Portfolio System (−)

The agent Euler equation and budget constraint were first-order approximated, cross-country differenced, and differenced with aggregate Euler and budget equations in (56) and (57). We begin from these equations.

Eliminating the real exchange rate from the cross-country differenced agent Euler equation (56) using the intermediate solution for the real exchange rate in (41), we obtain

$$\begin{aligned}&\theta\hat{W}^{i}_{i-jt}(\rho)+\beta R\,\mathrm{E}_t\Big[\tilde{C}_{i-jt+1}(\rho)\Big]-\left(\theta+\beta R\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QV}\right)\hat{W}^{i}_{i-jt}\\&\quad=\tilde{C}_{i-jt}(\rho)-\beta R\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\left(\eta^{(-)}_{QW}\hat{W}^{i}_{i-jt-1}+\eta^{(-)}_{QV}\hat{Y}_{i-jt}+\eta^{(-)}_{QV}\hat{V}^{i}_{i-jt}\right)\quad+O(\epsilon^2)\,,\end{aligned}\tag{B75}$$

where $\tilde{C}_{i-jt}(\rho)=\hat{C}_{i-jt}(\rho)-\hat{C}_{i-jt}$ denotes the difference between cross-country differenced agent and aggregate consumption, using the tilde notation to denote differences between agent and aggregate variables or parameters that integrate to zero.

Cross-country differenced aggregate real wealth $\hat{W}^{i}_{i-jt}$ now appears in (B75). A third equation, the law of motion for cross-country differenced aggregate real wealth in (38), must therefore join the Euler equation (B75) and budget constraint (57) to form the cross-country differenced agent system. We write the system in matrix form as

$$\begin{aligned}\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big]&=\boldsymbol{\mathcal{E}}^{(-)}_{1(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho)+\boldsymbol{\mathcal{E}}^{(-)}_{2(\rho)}\hat{Y}_{i-jt}\\&\quad+\boldsymbol{\mathcal{E}}^{(-)}_{3(\rho)}\hat{V}^{i}_{i-jt}(\rho)+\boldsymbol{\mathcal{E}}^{(-)}_{4(\rho)}\hat{V}^{i}_{i-jt}+O(\epsilon^2)\,,\end{aligned}\tag{B76}$$

where $\hat{\boldsymbol{Z}}_{i-jt}(\rho)=\Big[\hat{W}^{i}_{i-jt-1}(\rho)\quad\tilde{C}_{i-jt}(\rho)\quad\hat{W}^{i}_{i-jt-1}\Big]'$ denotes the (3×1) vector of cross-country differenced agent real wealth, cross-country differenced agent consumption in deviations from aggregate, and cross-country differenced aggregate real wealth, and where

$$\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}=\begin{bmatrix}1&0&-1\\\theta&\beta R&-\left(\theta+\beta R\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QW}\right)\\0&0&1\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{1(\rho)}=\begin{bmatrix}R&-1&-R\\0&1&-\beta R\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QW}\\0&0&\eta^{(-)}_{WW}\end{bmatrix},\quad\boldsymbol{\mathcal{E}}^{(-)}_{2(\rho)}=\begin{bmatrix}0\\-\beta R\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QY}\\\eta^{(-)}_{WY}\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{3(\rho)}=\begin{bmatrix}R\\0\\0\end{bmatrix},\quad\text{and}\quad\boldsymbol{\mathcal{E}}^{(-)}_{4(\rho)}=\begin{bmatrix}0\\-\beta R\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QV}\\\eta^{(-)}_{WV}\end{bmatrix}.$$

The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}$ is invertible if its determinant is non-zero. Therefore, we require that $\det\Big(\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}\Big)=\beta R\neq0$, which is always satisfied under Assumption 1.

Inverting $\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}$, we can rewrite the system in (B76) to obtain the system as it appears in (59), where matrices of partial elasticities are given by

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)W(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)W} \\ \mathcal{E}^{(-)}_{C(\rho)W(\rho)} & \mathcal{E}^{(-)}_{C(\rho)C(\rho)} & \tilde{\mathcal{E}}^{(-)}_{C(\rho)W} \\ \mathcal{E}^{(-)}_{WW(\rho)} & \mathcal{E}^{(-)}_{WC(\rho)} & \mathcal{E}^{(-)}_{WW} \end{bmatrix} = \begin{bmatrix} R & -1 & \eta^{(-)}_{WW} - R \\ -\frac{\theta}{\beta} & \frac{1+\theta}{\beta R} & \frac{\theta}{\beta} - \eta^{(-)}_{QW}\left(1-\eta^{(-)}_{WW}\right)\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right) \\ 0 & 0 & \eta^{(-)}_{WW} \end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)Y} \\ \tilde{\mathcal{E}}^{(-)}_{C(\rho)Y} \\ \mathcal{E}^{(-)}_{WY} \end{bmatrix} = \begin{bmatrix} \eta^{(-)}_{WY} \\ \left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\left(\eta^{(-)}_{QW}\eta^{(-)}_{WY} - \eta^{(-)}_{QY}\right) \\ \eta^{(-)}_{WY} \end{bmatrix}, \tag{B77}$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)V(\rho)} \\ \mathcal{E}^{(-)}_{C(\rho)V(\rho)} \\ \mathcal{E}^{(-)}_{WV(\rho)} \end{bmatrix} = \begin{bmatrix} R \\ -\frac{\theta}{\beta} \\ 0 \end{bmatrix}, \quad \text{and}$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)V} \\ \tilde{\mathcal{E}}^{(-)}_{C(\rho)V} \\ \mathcal{E}^{(-)}_{WV} \end{bmatrix} = \begin{bmatrix} \eta^{(-)}_{WV} \\ \left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\left(\eta^{(-)}_{QW}\eta^{(-)}_{WV} - \eta^{(-)}_{QV}\right) \\ \eta^{(-)}_{WV} \end{bmatrix}.$$

Before solving the cross-country differenced agent system, we decompose the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ into eigenvalues and eigenvectors and establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ is given by

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)} = \boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)}\boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(-)-1}_{Z(\rho)Z(\rho)} \tag{B78}$$

where $\boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)}$ is a matrix of eigenvectors of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ and where $\boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)}$ is a diagonal matrix of eigenvalues of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$. We write the matrix of eigenvectors as

$$\begin{aligned}\boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix} \nu^{(-)}_{W(\rho)W(\rho)} & \nu^{(-)}_{W(\rho)C(\rho)} & \nu^{(-)}_{W(\rho)W} \\ \nu^{(-)}_{C(\rho)W(\rho)} & \nu^{(-)}_{C(\rho)C(\rho)} & \nu^{(-)}_{C(\rho)W} \\ \nu^{(-)}_{WW(\rho)} & \nu^{(-)}_{WC(\rho)} & \nu^{(-)}_{WW} \end{bmatrix} \\ &= \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} \\ \lambda^{(-)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)} & \lambda^{(-)}_{ZC(\rho)} - \mathcal{E}^{(-)}_{WW} & \lambda^{(-)}_{ZW} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W} \\ 0 & 0 & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} \end{bmatrix},\end{aligned} \tag{B79}$$

and the diagonal matrix of eigenvalues as

$$\boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \lambda^{(-)}_{Z(\rho)W(\rho)} & 0 & 0 \\ 0 & \lambda^{(-)}_{Z(\rho)C(\rho)} & 0 \\ 0 & 0 & \lambda^{(-)}_{Z(\rho)W} \end{bmatrix}, \tag{B80}$$

where $\lambda^{(-)}_{Z(\rho)W} = \eta^{(-)}_{WW} = \lambda^{(-)}_{ZW}$, with $\lambda^{(-)}_{ZW}$ given in equation (B39), and where

$$\begin{aligned}\lambda^{(-)}_{Z(\rho)W(\rho)} = {} & \frac{1}{2}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} + \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big) \\ & + \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}}\end{aligned} \tag{B81}$$

and

$$\begin{aligned}\lambda^{(-)}_{Z(\rho)C(\rho)} = {} & \frac{1}{2}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} + \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big) \\ & - \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}}\,.\end{aligned} \tag{B82}$$

B.9.2 Stationarity of the Agent System $(-)$

We now establish uniqueness and stationarity for the cross-country differenced agent system. Following the same strategy as we did with the cross-country differenced aggregate system, we consider the neighborhood of values for θ around the knife-edge non-stationary case of $\theta = 0$ to establish the result. We state the following proposition.

Proposition B1 (Stationarity of the Cross-Country Differenced Agent System)**.** *The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ has two distinct real-valued eigenvalues, $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}$. Furthermore, $\lambda^{(-)}_{Z(\rho)W(\rho)}\Big|_{\theta=0} = 1/\beta$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}\Big|_{\theta=0} = 1$, and*

$$\partial\lambda^{(-)}_{Z(\rho)W(\rho)}/\partial\theta\,\Big|_{\theta=0} > 0 \quad \textit{and} \quad \partial\lambda^{(-)}_{ZC(\rho)}/\partial\theta\,\Big|_{\theta=0} < 0\,. \tag{B83}$$

The third eigenvalue $\lambda^{(-)}_{Z(\rho)C}$ equals the aggregate eigenvalue $\lambda^{(-)}_{ZC}$, which lies between zero and one if θ takes a small positive value (Proposition 5). Hence, there exists a $\theta > 0$ that yields a unique and stationary rational expectations solution to the cross-country differenced agent system in (59)*. All three eigenvalues are independent of ρ.*

To prove this statement, it suffices to show that the discriminant in (B81) and (B82) is positive. The discriminant, denoted $\Delta^{(-)}_{\rho}$, is given by

$$\Delta^{(-)}_{\rho} = \frac{1}{4}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}\,. \tag{B84}$$

Suppose $\theta > 0$. The squared term is always positive or zero, so it suffices to show that the second term is positive. We have

$$\mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} = \frac{\theta}{\beta} > 0\,, \tag{B85}$$

where we have used the partial elasticities in (B77) under Assumptions 1 and 2. Suppose $\theta = 0$. The second term is zero, $\mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} = 0$, but the squared term is positive, since $\mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big|_{\theta=0} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big|_{\theta=0} = 1/\beta - 1 > 0$ under Assumptions 1, so the discriminant

Table B3 – Agent Cross-Country Differenced Partial Elasticities at $\theta = 0$

Partial Elasticity	Value of Partial Elasticity Evaluated at $\theta = 0$		Derivative with Respect to θ Evaluated at $\theta = 0$	
	Expression	Sign	Expression	Sign
$\mathcal{E}^{(-)}_{W(\rho)W(\rho)}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{W(\rho)C(\rho)}$	-1	$-$	0	$\cdot$
$\mathcal{E}^{(-)}_{C(\rho)W(\rho)}$	0	$\cdot$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{C(\rho)C(\rho)}$	1	$+$	$1+\frac{1}{\beta}$	$+$

Notes. Column one lists the partial elasticities that appear in the cross-country differenced aggregate system (36). Columns two and four, respectively, give expressions for the partial elasticities and their derivatives with respect to θ, both evaluated at $\theta = 0$. Columns three and five, respectively, give the signs of the partial elasticities and their derivatives, both evaluated at $\theta = 0$. Signs are established under Assumptions 1 and 2.

$\Delta^{(-)}_{\rho}$ is again positive. Thus, $\theta \geq 0$ is sufficient to show that (B81) and (B82) are distinct and real-valued eigenvalues.

As in the cross-country differenced aggregate problem, the case where $\theta = 0$ serves as a helpful benchmark here for proving the further statements in Proposition B1. In Table B3, we evaluate at $\theta = 0$ the partial elasticities in (B77) and their derivatives with respect to θ.

Proposition B1 states that $\lambda^{(-)}_{Z(\rho)W(\rho)}$ evaluated at $\theta = 0$ exceeds one and $\lambda^{(-)}_{Z(\rho)C(\rho)}$ evaluated at $\theta = 0$ equals one. Using Table B3 to evaluate (B81) and (B82) at $\theta = 0$,

$$\lambda^{(-)}_{Z(\rho)W(\rho)}\Big|_{\theta=0} = \mathcal{E}^{(-)}_{WW}\Big|_{\theta=0} > 1 \quad \text{and} \quad \lambda^{(-)}_{Z(\rho)C(\rho)}\Big|_{\theta=0} = \mathcal{E}^{(-)}_{CC}\Big|_{\theta=0} = 1\,. \tag{B86}$$

Proposition B1 further states that the partial derivative of $\lambda^{(-)}_{Z(\rho)W(\rho)}$ with respect to θ evaluated at $\theta = 0$ is positive, and the partial derivative of $\lambda^{(-)}_{Z(\rho)C(\rho)}$ with respect to θ evaluated at $\theta = 0$ is negative. To establish this result, we differentiate the eigenvalues with respect to θ,

$$\begin{aligned}
\frac{\partial \lambda^{(-)}_{\pm}}{\partial \theta} &= \frac{1}{2}\left(\frac{\partial \mathcal{E}^{(-)}_{C(\rho)C(\rho)}}{\partial \theta} + \frac{\partial \mathcal{E}^{(-)}_{W(\rho)W(\rho)}}{\partial \theta}\right) \\
&\quad \pm \frac{1}{2}\left[\frac{1}{4}\left(\mathcal{E}^{(-)}_{C(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\right)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}\right]^{-\frac{1}{2}} \\
&\quad \times \left[\frac{1}{2}\left(\mathcal{E}^{(-)}_{C(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\right)\left(\frac{\partial \mathcal{E}^{(-)}_{C(\rho)C(\rho)}}{\partial \theta} - \frac{\partial \mathcal{E}^{(-)}_{W(\rho)W(\rho)}}{\partial \theta}\right)\right. \\
&\qquad\qquad \left. + \frac{\partial \mathcal{E}^{(-)}_{W(\rho)C(\rho)}}{\partial \theta}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\frac{\partial \mathcal{E}^{(-)}_{C(\rho)W(\rho)}}{\partial \theta}\right],
\end{aligned} \tag{B87}$$

where the "+" case corresponds to $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and the "−" case corresponds to $\lambda^{(-)}_{Z(\rho)C(\rho)}$. Using Table B3, we evaluate (B87) at $\theta = 0$ to obtain

$$\left.\frac{\partial \lambda^{(-)}_{Z(\rho)W(\rho)}}{\partial \theta}\right|_{\theta=0} = \frac{1}{1-\beta} - \frac{1}{\beta} \quad \text{and} \quad \left.\frac{\partial \lambda^{(-)}_{Z(\rho)C(\rho)}}{\partial \theta}\right|_{\theta=0} = \frac{1}{\beta} - \frac{1}{1-\beta}\,. \tag{B88}$$

The derivatives with respect to $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}$ are positive and negative, respectively, if $1/2 < \beta < 1$, which is satisfied under Assumption 1.

Finally, Proposition B1 states that the three eigenvalues of the matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ are independent of ρ. This result follows straightforwardly from the definitions of the eigenvalues in (B40), (B81), and (B82), together with the definitions of the partial elasticities in (B36) and (B77).

In view of the eigenvalues at $\theta = 0$ in (B44) and (B86), and the derivatives at $\theta = 0$ in (B49), (B88) and (B88) under Assumptions 1 and 2, a small positive value for θ pushes $\lambda^{(-)}_{Z(\rho)C(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C}$ below one while pushing $\lambda^{(-)}_{Z(\rho)W(\rho)}$ further above one, thus satisfying the conditions in Blanchard and Kahn (1980) for a unique and stationary rational expectations solution to the cross-country differenced agent system in (59).

B.9.3 Intermediate Agent Non-Portfolio Solution (−)

We now derive intermediate solutions for agent real wealth and consumption in terms of lagged agent and aggregate state variables, exogenous variables and parameters, and the endogenous agent and aggregate portfolio valuation effects. Using the eigendecomposition in (59), we re-write the agent cross-country differenced system in (B76) as

$$\begin{aligned} &\boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big] = \boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho) \\ &\quad + \boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\Big(\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}\hat{V}^{i}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}\hat{V}^{i}_{i-jt}\Big) + O(\epsilon^2)\,, \end{aligned} \tag{B89}$$

and from (B89) we extract the equation associated with the unstable eigenvalue $\lambda^{(-)}_{Z(\rho)W(\rho)}$,

$$\begin{aligned} &\mathrm{E}_t\Big[\hat{Z}_{i-jt+1}(\rho)\Big] = \lambda^{(-)}_{Z(\rho)W(\rho)}\hat{Z}_{i-jt}(\rho) \\ &+ \Big[\Big(\lambda^{(-)}_{Z(\rho)W} - \mathcal{E}^{(-)}_{W(\rho)W}\Big)\eta^{(-)}_{WY} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\mathcal{E}^{(-)}_{W(\rho)Y} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{\mathcal{E}}^{(-)}_{C(\rho)Y}\Big]\hat{Y}_{i-jt} \\ &+ \Big[\Big(\lambda^{(-)}_{Z(\rho)W} - \mathcal{E}^{(-)}_{W(\rho)W}\Big)\eta^{(-)}_{WV} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\mathcal{E}^{(-)}_{W(\rho)V} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{\mathcal{E}}^{(-)}_{C(\rho)V}\Big]\hat{V}^{i}_{i-jt} \\ &+ \Big[\Big(\lambda^{(-)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)V(\rho)}\Big]\tilde{V}^{i}_{i-jt}(\rho) + O(\epsilon^2)\,, \end{aligned} \tag{B90}$$

where $\hat{Z}_{i-jt+1}(\rho) = \Big(\lambda^{(-)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big)\hat{W}^{i}_{i-jt}(\rho) - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{C}_{i-jt+1}(\rho)$.

Because $\lambda^{(-)}_{Z(\rho)W(\rho)}$ is greater than one, the left-hand side of (B51) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\begin{aligned}\tilde{C}_{i-jt}(\rho) &= \eta^{(-)}_{C(\rho)W(\rho)}\tilde{W}^i_{i-jt-1}(\rho) + \eta^{(-)}_{C(\rho)C(\rho)}\tilde{V}^i_{i-jt}(\rho)\\ &\quad + \tilde{\eta}^{(-)}_{C(\rho)V}\hat{V}^i_{i-jt} + \tilde{\eta}^{(-)}_{C(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{B91}$$

and using (B91) to eliminate $\tilde{C}_{i-jt}(\rho)$ from the cross-country differenced agent budget constraint in (57), we obtain

$$\begin{aligned}\tilde{W}^i_{i-jt}(\rho) &= \eta^{(-)}_{W(\rho)W(\rho)}\tilde{W}^i_{i-jt-1}(\rho) + \eta^{(-)}_{W(\rho)V(\rho)}\tilde{V}^i_{i-jt}(\rho)\\ &\quad + \tilde{\eta}^{(-)}_{W(\rho)V}\hat{V}^i_{i-jt} + \tilde{\eta}^{(-)}_{W(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,.\end{aligned} \tag{B92}$$

Writing the intermediate solutions (B91) and (B92) in more compact matrix form, we obtain (61), where the semi-partial elasticity matrices are given by

$$\begin{aligned}
\boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)} &= \begin{bmatrix}\eta^{(-)}_{W(\rho)W(\rho)}\\ \eta^{(-)}_{C(\rho)W(\rho)}\end{bmatrix} = \begin{bmatrix}\lambda^{(-)}_{Z(\rho)C(\rho)}\\ \mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\end{bmatrix},\\
\boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)} &= \begin{bmatrix}\eta^{(-)}_{W(\rho)V(\rho)}\\ \eta^{(-)}_{C(\rho)V(\rho)}\end{bmatrix} = \begin{bmatrix}\frac{1-\theta}{\beta} - \frac{\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\\ \frac{\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\end{bmatrix},\\
\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V} &= \begin{bmatrix}\tilde{\eta}^{(-)}_{W(\rho)V}\\ \tilde{\eta}^{(-)}_{C(\rho)V}\end{bmatrix} = \begin{bmatrix}(\bar{\sigma} - \sigma(\rho))\frac{\eta^{(-)}_{QV} - \eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\\ (\sigma(\rho) - \bar{\sigma})\frac{\eta^{(-)}_{QV} - \eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\end{bmatrix}, \quad \text{and}\\
\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y} &= \begin{bmatrix}\tilde{\eta}^{(-)}_{W(\rho)Y}\\ \tilde{\eta}^{(-)}_{C(\rho)Y}\end{bmatrix} = \begin{bmatrix}(\bar{\sigma} - \sigma(\rho))\frac{\eta^{(-)}_{QY} - \eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\\ (\sigma(\rho) - \bar{\sigma})\frac{\eta^{(-)}_{QY} - \eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\end{bmatrix}.
\end{aligned} \tag{B93}$$

Next, we follow a similar procedure to derive an intermediate solution to the agent's cross-country summed non-portfolio system.

B.9.4 The Agent Non-Portfolio System (+)

We use the first-order approximate agent and aggregate Euler equations in (28) and (29) and the first-order approximate agent and aggregate budget constraints in (30) to derive the cross-country summed agent system in deviations from aggregates. This system is simpler than the cross-country differenced agent system, first because the real exchange rate drops out of the system and second because the system has only one predetermined state variable, agent real wealth. Aggregate real wealth does not enter because its cross-country sum is zero with assets in zero net supply.

Using first-order approximate agent and aggregate Euler equations, we obtain the first-order approximate cross-country summed agent Euler equation in deviations from aggregates,

$$\begin{aligned}\theta\Big(\hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\Big) + \beta R\,\mathrm{E}_t\Big[\Big(\hat{C}_{i+jt+1}(\rho) - \hat{C}_{i+jt+1}\Big)\Big] \\ = \Big(\hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\Big) - \frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Y}_{i+jt} + O(\epsilon^2)\,,\end{aligned} \tag{B94}$$

where the subscript $i+j$ on agent variables denotes cross-country sums between agents with identical coefficients of relative risk aversion. Similarly, using first-order approximate agent and aggregate budget constraints, we obtain the first-order approximate cross-country summed agent budget constraint in deviations from aggregates,

$$\begin{aligned}\Big(\hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\Big) = R\Big(\hat{W}^i_{i+jt-1}(\rho) - \hat{W}^i_{i+jt-1}\Big) \\ - \Big(\hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\Big) + R\Big(\hat{V}^i_{i+jt}(\rho) - \hat{V}^i_{i+jt}\Big) + O(\epsilon^2)\,,\end{aligned} \tag{B95}$$

where we have introduced a new variable $\hat{V}^i_{i+jt}(\rho)$ in the budget constraint, which we call the cross-country summed agent portfolio valuation effect and define as

$$\hat{V}^i_{i+jt}(\rho) = B^i_{ii+ji}(\rho)\hat{R}^i_{i-jt}\,, \quad \text{where} \quad B^i_{ii+ji}(\rho) = B^i_{ii}(\rho) + B^i_{ji}(\rho)\,. \tag{B96}$$

Unlike in the cross-country differenced agent problem, neither the cross-country summed Euler equation in (B94) nor the cross-country summed budget constraint in (B95) depends on the real exchange rate.

The absence of the real exchange rate allows use to write the cross-country summed agent Euler equation and budget constraint directly in deviations from aggregates. Continuing to use tildes to indicate the vanishing integral property, we have

$$\begin{aligned}\tilde{W}^i_{i+jt}(\rho) = \hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\,, \quad \tilde{C}_{i+jt}(\rho) = \hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\,, \\ \text{and} \quad \tilde{V}^i_{i+jt}(\rho) = \hat{V}^i_{i+jt}(\rho) - \hat{V}^i_{i+jt}\,,\end{aligned} \tag{B97}$$

where, due to our assumption that bonds are in zero net supply, cross-country summed aggregate real wealth and portfolio valuation effects equal zero, $\hat{W}^i_{i+jt} = \hat{V}^i_{i+jt} = 0$. To maintain consistency in our notation, however, we still use the tilde to denote cross-country summed agent real wealth and portfolio valuation effects in deviations from aggregates.

The two equations (B94) and (B95) form the cross-country summed agent system, which we write in matrix form as

$$\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}\,\mathrm{E}_t\Big[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\Big] = \boldsymbol{\mathcal{E}}^{(+)}_{1(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho) + \tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{2(\rho)}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{3(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B98}$$

where $\tilde{\boldsymbol{Z}}_{i+jt}(\rho) = \left[\tilde{W}^i_{i+jt-1}(\rho) \quad \tilde{C}_{i+jt}(\rho)\right]'$ denotes the (2×1) vector of cross-country summed agent real wealth and consumption in deviation from aggregate, where

$$\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)} = \begin{bmatrix} 1 & 0 \\ \frac{\theta}{\beta R} & 1 \end{bmatrix}, \qquad \boldsymbol{\mathcal{E}}^{(+)}_{1(\rho)} = \begin{bmatrix} R & -1 \\ 0 & \frac{1}{\beta R} \end{bmatrix},$$
$$\tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{2(\rho)} = \begin{bmatrix} 0 \\ -\frac{1}{\beta R}\frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\rho}\right) \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\mathcal{E}}^{(+)}_{3(\rho)} = \begin{bmatrix} R \\ 0 \end{bmatrix}. \tag{B99}$$

The matrix $\mathcal{E}^{(+)}_{0(\rho)}$ is invertible if its determinant is non-zero. We have

$$\det\left(\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}\right) = 1 \neq 0\,, \tag{B100}$$

so $\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}$ is invertible, and we rewrite the cross-country summed agent system as

$$\mathrm{E}_t\left[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\right] = \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho) + \tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B101}$$

where

$$\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)W(\rho)} & \mathcal{E}^{(+)}_{W(\rho)C(\rho)} \\ \mathcal{E}^{(+)}_{C(\rho)W(\rho)} & \mathcal{E}^{(+)}_{C(\rho)C(\rho)} \end{bmatrix} = \begin{bmatrix} R & -1 \\ -\frac{\theta}{\beta} & \frac{1+\theta}{\beta R} \end{bmatrix},$$
$$\tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{Z(\rho)Y} = \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)Y} \\ \tilde{\mathcal{E}}^{(+)}_{C(\rho)Y} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{\beta R}\frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\rho}\right) \end{bmatrix}, \quad \text{and} \tag{B102}$$
$$\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)} = \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)V(\rho)} \\ \mathcal{E}^{(+)}_{C(\rho)V(\rho)} \end{bmatrix} = \begin{bmatrix} R \\ -\frac{\theta}{\beta} \end{bmatrix}.$$

Before solving the cross-country summed agent system, we decompose the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ into eigenvalues and eigenvectors and establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ is given by

$$\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} = \boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)} \tag{B103}$$

where $\boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)}$ is a matrix of eigenvectors of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ and where $\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}$ is a diagonal matrix of eigenvalues of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$,

$$\boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \nu^{(+)}_{W(\rho)W(\rho)} & \nu^{(+)}_{W(\rho)C(\rho)} \\ \nu^{(+)}_{C(\rho)W(\rho)} & \nu^{(+)}_{C(\rho)C(\rho)} \end{bmatrix} = \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)C(\rho)} & \mathcal{E}^{(+)}_{W(\rho)C(\rho)} \\ \lambda^{(+)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)} & \lambda^{(+)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)} \end{bmatrix}, \quad \text{and} \tag{B104}$$

$$\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \lambda^{(+)}_{Z(\rho)W(\rho)} & 0 \\ 0 & \lambda^{(+)}_{Z(\rho)C(\rho)} \end{bmatrix},$$

where

$$\lambda^{(+)}_{Z(\rho)W(\rho)} = \frac{1}{2}\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} + \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\right) + \sqrt{\frac{1}{4}\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\right)^2 + \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)W(\rho)}} \tag{B105}$$

and

$$\lambda^{(+)}_{Z(\rho)C(\rho)} = \frac{1}{2}\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} + \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\right) - \sqrt{\frac{1}{4}\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\right)^2 + \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)W(\rho)}}. \tag{B106}$$

B.9.5 Stationarity of the Agent System (+)

We now establish uniqueness and stationarity for the cross-country differenced agent system. Stationarity of the system follows straightforwardly from a comparison of the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ in (B102) with the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ in (B77). We state the following proposition.

Proposition B2 (Stationarity of the Cross-Country Summed Agent System)**.** *The eigenvalues of* $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ *are given by*

$$\lambda^{(+)}_{Z(\rho)W(\rho)} = \lambda^{(-)}_{Z(\rho)W(\rho)} \quad \textit{and} \quad \lambda^{(+)}_{Z(\rho)C(\rho)} = \lambda^{(-)}_{Z(\rho)C(\rho)}\,. \tag{B107}$$

From Proposition B1, there exists a $\theta > 0$ *that yields a unique and stationary rational expectations solution to the cross-country summed agent system in* (B101)*.*

Let $\left[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\right]_{33}$ denote the (2×2) submatrix of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ formed by deleting the third row and third column from $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$. Then, by inspection, $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} = \left[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\right]_{33}$, and the eigenvalues of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ equal the eigenvalues of the submatrix $\left[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\right]_{33}$. That is, $\lambda^{(+)}_{Z(\rho)W(\rho)} = \lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(+)}_{Z(\rho)C(\rho)} = \lambda^{(-)}_{Z(\rho)C(\rho)}$. Thus, the stationarity and uniqueness conditions for the cross-country summed and differenced agent systems are the same, and satisfied under Assumptions 1 and 2 as shown in Proposition B1.

B.9.6 Intermediate Agent Non-Portfolio Solution (+)

We now derive intermediate solutions for cross-country summed agent real wealth and consumption in deviation from aggregate, written in terms of lagged agent state variables, exogenous variables and parameters, and the endogenous agent portfolio valuation effect. Using the eigendecomposition in (B103), we re-write the system in (B76) as

$$\begin{aligned}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\,\mathrm{E}_t\Big[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\Big] &= \boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho)\\ &+\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\Big(\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt}+\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)}\hat{V}^{i}_{i+jt}(\rho)\Big)+O(\epsilon^2)\,,\end{aligned} \tag{B108}$$

and from (B108) we extract the equation associated with the unstable eigenvalue $\lambda^{(+)}_{Z(\rho)W(\rho)}$,

$$\begin{aligned}\mathrm{E}_t\Big[\tilde{Z}_{i+jt+1}(\rho)\Big] &= \lambda^{(+)}_{Z(\rho)W(\rho)}\tilde{Z}_{i+jt}(\rho)-\mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt}\\ &+\Big[\Big(\lambda^{(+)}_{Z(\rho)C(\rho)}-\mathcal{E}^{(+)}_{W(\rho)W(\rho)}\Big)\mathcal{E}^{(+)}_{W(\rho)W(\rho)}-\mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)V(\rho)}\Big]\tilde{V}^{i}_{i+jt}(\rho)+O(\epsilon^2)\,,\end{aligned} \tag{B109}$$

where $\tilde{Z}_{i+jt+1}(\rho)=\Big(\lambda^{(+)}_{Z(\rho)W(\rho)}-\mathcal{E}^{(+)}_{W(\rho)W(\rho)}\Big)\tilde{W}^{i}_{i+jt}(\rho)-\mathcal{E}^{(+)}_{W(\rho)W(\rho)}\tilde{C}_{i+jt+1}(\rho)$.

Because $\lambda^{(+)}_{Z(\rho)W(\rho)}$ is greater than one, the left-hand side of (B109) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\tilde{C}_{i+jt}(\rho)=\eta^{(+)}_{C(\rho)W(\rho)}\tilde{W}^{i}_{i+jt-1}(\rho)+\tilde{\eta}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt}+\eta^{(+)}_{C(\rho)V(\rho)}\tilde{V}^{i}_{i+jt}(\rho)+O(\epsilon^2)\,, \tag{B110}$$

and using (B110) to eliminate $\tilde{C}_{i+jt}(\rho)$ from the cross-country summed agent budget constraint in (B95), we obtain

$$\tilde{W}^{i}_{i+jt}(\rho)=\eta^{(+)}_{W(\rho)W(\rho)}\tilde{W}^{i}_{i+jt-1}(\rho)+\tilde{\eta}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt}+\eta^{(+)}_{W(\rho)V(\rho)}\tilde{V}^{i}_{i+jt}(\rho)+O(\epsilon^2)\,, \tag{B111}$$

where the cross-country summed agent wealth and consumption semi-partial elasticity matrices are given by

$$\begin{aligned}\boldsymbol{\eta}^{(+)}_{Z(\rho)W(\rho)} &= \begin{bmatrix}\eta^{(+)}_{W(\rho)W(\rho)}\\ \eta^{(+)}_{C(\rho)W(\rho)}\end{bmatrix}=\begin{bmatrix}\lambda^{(+)}_{Z(\rho)C(\rho)}\\ \mathcal{E}^{(+)}_{W(\rho)W(\rho)}-\lambda^{(+)}_{Z(\rho)C(\rho)}\end{bmatrix},\\ \tilde{\boldsymbol{\eta}}^{(+)}_{Z(\rho)Y} &= \begin{bmatrix}\tilde{\eta}^{(+)}_{W(\rho)Y}\\ \tilde{\eta}^{(+)}_{C(\rho)Y}\end{bmatrix}=\begin{bmatrix}\frac{\tilde{\mathcal{E}}^{(+)}_{C(\rho)Y}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\\ -\frac{\tilde{\mathcal{E}}^{(+)}_{C(\rho)Y}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\end{bmatrix},\quad\text{and}\\ \boldsymbol{\eta}^{(+)}_{Z(\rho)V(\rho)} &= \begin{bmatrix}\eta^{(+)}_{W(\rho)V(\rho)}\\ \eta^{(+)}_{C(\rho)V(\rho)}\end{bmatrix}=\begin{bmatrix}\frac{1-\theta}{\beta}-\frac{\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)}-\lambda^{(+)}_{Z(\rho)C(\rho)}\Big)\mathcal{E}^{(-)}_{W(\rho)V(\rho)}-\mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\\ \frac{\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)}-\lambda^{(+)}_{Z(\rho)C(\rho)}\Big)\mathcal{E}^{(-)}_{W(\rho)V(\rho)}-\mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\end{bmatrix}.\end{aligned} \tag{B112}$$

Writing the intermediate solutions in (B110) and (B111) in matrix form, we have

$$\begin{bmatrix} \tilde{W}^i_{i+jt}(\rho) \\ \tilde{C}_{i+jt}(\rho) \end{bmatrix} = \boldsymbol{\eta}^{(+)}_{Z(\rho)W(\rho)} \tilde{W}^i_{i+jt-1}(\rho) + \tilde{\boldsymbol{\eta}}^{(+)}_{Z(\rho)Y} \hat{Y}_{i+jt} + \boldsymbol{\eta}^{(+)}_{Z(\rho)V(\rho)} \tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2) \,. \tag{B113}$$

As with the cross-country differenced solution in (61), the cross-country summed solution in (B113) is intermediate in that it depends on the endogenous portfolio valuation effect $\tilde{V}^i_{i+jt}(\rho)$. This effect in turn depend on the endogenous steady-state cross-country sum of agent portfolio holdings, for which we solve next.

B.10 The Agent Portfolio Problem

We now solve for zero-order approximate cross-country differenced and summed agent portfolio holdings. These solutions then combine to yield country-specific solutions for the agent's holdings of the domestic and international bond.

To derive cross-country differenced agent portfolio holdings, we begin from equation (62). Using the real exchange rate in (41), the definition of the cross-country differenced agent portfolio valuation effect in (58), and the first-order approximate intermediate solutions for cross-country differenced agent consumption in (61), we obtain the intermediate solution in (63). The collection of parameters $\zeta^{(-)}_{B(\rho)}$ in (63) is given by

$$\begin{aligned} \zeta^{(-)}_{B(\rho)} = & \frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}} \left[\left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right) \eta^{(-)}_{QY} + \frac{\eta^{(-)}_{QW} \eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right] \\ & - \frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}} \frac{\eta^{(-)}_{CY} - \eta^{(-)}_{QY} \frac{\omega}{\bar{\rho}}}{\eta^{(-)}_{CV} - \eta^{(-)}_{QV} \frac{\omega}{\bar{\rho}}} \left[\left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right) \eta^{(-)}_{QV} + \frac{\eta^{(-)}_{QW} \eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right] . \end{aligned} \tag{B114}$$

To derive cross-country summed agent portfolio holdings, we begin from equation (64). Using the real exchange rate in (41), the definition of the cross-country summed agent portfolio valuation effect in (B96), and the first-order approximate intermediate solutions for cross-country summed agent consumption in (B113), we obtain the intermediate solution in (65). The collection of parameters $\zeta^{(+)}_{B(\rho)}$ in (65) is given by

$$\zeta^{(+)}_{B(\rho)} = \frac{\bar{\rho}}{\omega} \frac{1}{\eta^{(+)}_{C(\rho)V(\rho)}} \left(1 - \frac{1}{(1-\theta)\lambda^{(-)}_{Z(\rho)W(\rho)}} \right) . \tag{B115}$$

The solutions in equations (63) and (65) are intermediate because the cross-country differenced real return appears on the right-hand side of each. Final solutions obtain straightforwardly by eliminating the cross-country differenced real return using the first-order approximate general solution in (43). For brevity, we omit this derivation here, noting that (63) and (65) are sufficient to establish our main result for agent portfolio

Table B4 – Agent Semi-Partial Elasticities at $\theta = 0$.

Semi-Partial Elasticity	Evaluation at $\theta = 0$	
	Expression	Sign
$\eta^{(-)}_{C(\rho)V(\rho)}$	$\frac{1-\beta}{\beta}$	$+$
$\eta^{(+)}_{C(\rho)V(\rho)}$	$\frac{1-\beta}{\beta}$	$+$

Notes. The table shows semi-partial elasticities from the cross-country differenced and summed agent non-portfolio problems, evaluated at $\theta = 0$. Column two give an expression for each elasticity, and column three gives its sign under Assumptions 1 and 2.

holdings in Proposition (10). For use later, we evaluate two semi-partial elasticities that appear in these solutions at $\theta = 0$ and report them in Table B4.

B.10.1 Proof of Proposition 10

Proposition 10 establishes sufficient conditions for international portfolio bias in agent portfolio holdings for a subset of risk-tolerant agents. These conditions simultaneously satisfy the conditions for domestic bias in aggregate portfolio holdings in Proposition 8 and for negative expected cross-country differenced real returns in Proposition 9.

We begin with equation (66) for agent holdings of the domestic bond in country i. Using the intermediate solutions for cross-country differenced and summed agent holdings of this bond in (63) and (65), respectively, we obtain

$$\tilde{B}^i_{ii}(\rho) = \frac{1}{2}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right) \frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\zeta^{(-)}_{B(\rho)}\hat{Y}_{i-jt+1} + \zeta^{(+)}_{B(\rho)}\hat{Y}_{i+jt+1}\right)\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{B116}$$

from which a final solution in terms of model parameters can be obtained directly using first-order approximate realized real returns in (42).

In the proof of Proposition (9) we established $\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right] \leq 0$ and $\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\right] <$ 0 under Assumptions 1 and 2. To establish the sign of $\tilde{B}^i_{ii}(\rho)$, it remains to establish the signs of $\zeta^{(-)}_{B(\rho)}$ and $\zeta^{(+)}_{B(\rho)}$.

We first establish the sign of $\zeta^{(-)}_{B(\rho)}$ assuming $\theta = 0$. Splitting the expression in (B114) into four terms, we have

$$\zeta^{(-)}_{B(\rho)} = \zeta^{(-)(T_1)}_{B(\rho)}\left(\zeta^{(-)(T_2)}_{B(\rho)} + \zeta^{(-)(T_3)}_{B(\rho)}\zeta^{(-)(T_4)}_{B(\rho)}\right), \tag{B117}$$

where

$$\zeta_{B(\rho)}^{(-)(T_1)} = \frac{1}{\eta_{C(\rho)V(\rho)}^{(-)}}\,,$$

$$\zeta_{B(\rho)}^{(-)(T_2)} = \left(1 - \frac{1}{\lambda_{Z(\rho)W(\rho)}^{(-)}}\right)\eta_{QY}^{(-)} + \frac{1}{\lambda_{Z(\rho)W(\rho)}^{(-)}}\eta_{QW}^{(-)}\eta_{WY}^{(-)}\,,$$

$$\zeta_{B(\rho)}^{(-)(T_3)} = -\frac{\eta_{C}^{(-)}oo - \bar{\sigma}\eta_{QY}^{(-)}}{\eta_{CV}^{(-)} - \bar{\sigma}\eta_{QV}^{(-)}}\,, \quad \text{and} \tag{B118}$$

$$\zeta_{B(\rho)}^{(-)(T_4)} = \left(1 - \frac{1}{\lambda_{Z(\rho)W(\rho)}^{(-)}}\right)\eta_{QV}^{(-)} + \frac{1}{\lambda_{Z(\rho)W(\rho)}^{(-)}}\eta_{QW}^{(-)}\eta_{WV}^{(-)}\,.$$

Using Table B2, Table B4, and the expression for $\lambda_{Z(\rho)W}^{(-)}\Big|_{W=0}$ in Proposition B1, we obtain

$$\zeta_{B(\rho)}^{(-)(T_1)}\Big|_{\theta=0} = \frac{\beta}{1-\beta}\Big|_{\theta=0} > 0\,,$$

$$\zeta_{B(\rho)}^{(-)(T_2)}\Big|_{\theta=0} = (1-\beta)\left(1 + \beta\frac{\left(1-\phi^2\right)(1-\bar{\sigma})}{1-(1-\bar{\sigma})\phi^2}\right) > 0\,,$$

$$\zeta_{B(\rho)}^{(-)(T_3)}\Big|_{\theta=0} = -\phi(1-\bar{\sigma})\frac{(1-\beta)^2}{\beta}\frac{1-(1-\bar{\sigma})\phi^2}{1-\phi} < 0\,, \quad \text{and} \tag{B119}$$

$$\zeta_{B(\rho)}^{(-)(T_4)}\Big|_{\theta=0} = -\frac{(1-\beta)^2}{\beta}\frac{\phi^2}{1-\phi}\left(1 + \frac{1-\phi^2-(1-\beta)(1-\phi)}{1-\phi^2}\right) < 0\,.$$

Next, we establish the sign of $\zeta_{B(\rho)}^{(+)}$ assuming $\theta = 0$. Using Table B2 and the expression for the eigenvalue $\lambda_{Z(\rho)W}^{(-)}$ in Proposition B1, we obtain $\zeta_{B(\rho)}^{(+)}\Big|_{\theta=0} = \beta\bar{\rho}/\omega > 0$ under Assumptions 1 and 2. By continuity, the signs of $\zeta_{B(\rho)}^{(-)}$ and $\zeta_{B(\rho)}^{(+)}$ are preserved for small positive values of θ.

Returning to equation (B116), we conclude that $\tilde{B}_{ii}^{i}(\rho)$ is negative for any $\rho < \bar{\rho}/\omega$ under Assumptions 1–3, and that $\tilde{B}_{ii}^{i}(\rho)$ approaches negative infinity as ρ approaches zero from above,

$$\lim_{\rho\downarrow 0}\tilde{B}_{ii}^{i}(\rho) = -\infty\,. \tag{B120}$$

Thus, recalling that $\tilde{B}_{ii}^{i}(\rho) \equiv B_{ii}^{i}(\rho) - B_{ii}^{i}$, there exists a $\rho^* \in (0, \bar{\rho}/\omega)$ such that $B_{ii}^{i}(\rho) < 0$ for any $\rho < \rho^*$ and any finite value of B_{ii}^{i}.

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